Math 113(2) - Comments for HW2

Seewoo Lee

September 5, 2018

Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use LaTeX.
4. For questions that requires proofs, I almost not give any partial credits.

Section 2, Problem 10

Commutativity holds (+5 points), but associativity doesn't holds (+5 points). You have to give at least one counterexample, such as $(0 * 0) * 1 \neq 0 * (0 * 1)$. If not, I deducted 1 point.

Section 3, Problem 16a

You have to clearly write the definition of $*$ that makes ϕ as a group isomorphism.

- Define $x * y$ as $x * y := x + y - 1$. (If you don't, -2 points)
- $\phi : (\mathbb{Z}, +) \to (\mathbb{Z}, *)$ defined by $\phi(n) = n + 1$ is a homomorphism, hence isomorphism. (If you don't, -2 points)
- Identity of $(\mathbb{Z}, *)$ is 1. (If you don't, -2 points)

Section 3, Problem 33

There's a little mistake here - $\mathbb{C}$ and H aren't group with a multiplication, because of the existence of $0 \in \mathbb{C}$ and $O \in H$. However, if we exclude zeros from them, it become a group. So the correct statement of (b) should be that $(\mathbb{C}\backslash\{0\}, \cdot)$ and $(H\backslash\{O\}, \cdot)$ are isomorphic. (Actually, both $\mathbb{C}$ and H are *rings*, which have both addition and multiplication). Anyway, you have to show the followings:

- ϕ is injective and surjective, hence bijective. (This is trivial, but at least you have to mention it. 1 point for each.)
- For (a), ϕ is an additive homomorphism: $\phi(z+w)=\phi(zw)$. (4 points)
- For (b), ϕ is a multiplicative homomorphism: $\phi(zw) = \phi(z)\phi(w)$. (4 points)