Math 113(2) - Comments for HW3

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Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use $\LaTeX$.
4. For questions that requires proofs, I almost not give any partial credits.

Section 4, Problem 30

- a) 3 points. No partial credits.
- b) 3 points. Both 1 and -1 are left identities, and a right inverse of $a \in \mathbb{R}^*$ is $1/|a|$ or $-1/|a|$, which depends on the choice of the left identity. You should have choose consistently, not just writing it as ± 1 and $\pm 1/|a|$. Also, for any cases, $1/a$ can't be a right inverse of a. (-1 point).
- c) 3 points. There are several possible answers for this:
  - Both 1 and -1 can't be a right identity,
  - Left inverse is not unique.

  However, you should be careful about *left* and *right* things. If you didn't, I deducted 1 point.
- d) I give 1 point for every answer, since there are many possible answers for this, such as
  - If you want to weaken axioms for a group in terms of one-sided identity and one-sided inverse, both side should be chosen in a same way. For example, existence of left identity and left inverse is enough to weaken axioms. However, you can use axioms like existence of left identity and right inverse, since it may not define a group, and this exercise is a good example for it. (This is the most reasonable answer)

- Checking the existence of identity is not sufficient, and we also have to check uniqueness of ti.
- Identity should be both left and right sided.
- It is not always easy to show that some set equipped with a binary operation is a group.

Section 5, Problem 16

- a) 2 points. The only possible identity for the addition is zero function $f \equiv 0$, which does not contained in the set $\widetilde{F}$. Also, the set $\widetilde{F}$ is not closed under the addition, since if $f, g \in \widetilde{F}$, then $f(1) = g(1) = 1$ so $(f+g)(1) = f(1) + g(1) = 2 \neq 1$.

- b) 8 points. You have to show that $\widetilde{F}$ is closed under multiplication, it contains the identity element, and inverse exists for each element in $\widetilde{F}$. When you say about inverse, you should mention that $(1/f)(1) = 1/f(1) = 1$ for $f \in \widetilde{F}$. (-1 point if you didn't mention it). Also, -2 points for each part omitted.

Section 5, Problem 45

The *only if* part, which is fairly easy, is 2 points and *if* part is 8 points. You should show the both directions. For the *if* part, you have to show that

1. There exists an identity (2 points),
2. every elements in H has an inverse (3 points),
3. and H is closed under the operation.

It is really important to show in the above order. For example, you can't say anything about the existence of the inverse before you show the existence of the identity element. I deducted some points if your order is different from the above. Also, when you show the existence of the identity element, you have to use nonemptyness of H to *choose* an element a in H. However, there are very few people who mentioned about this, so I didn't deduct any point for this.