Math 113(2) - Comments for HW6

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Some general comments:

1. **DO NOT CHEAT!**
2. Please use staplers or clips, not just fold the left-upper corner of papers!
3. Try to write well! - maybe this will be harder than the first one...
4. If you can, try to use LaTeX.
5. For questions that requires proofs, I almost not give any partial credits.

Section 9, Problem 29

If every element of H are even, we are done. If it isn't, there exists an odd permutation $a \in H$ and the map $H \to H, x \mapsto ax$ gives a bijection between the set even permutations and odd permutations in H. (You have to show this, otherwise -2 point). Some of you just pick an arbitrary $a \in H$ and *show* that $x \mapsto ax$ gives a bijection between even and odd permutations of H. This is only possible when a is an odd permutation, so you need to specify about it. Otherwise, -2 point. Note that you can't take $a = (12)$, since H might not contain a transposition. (Consider $H = \langle (12)(34)(56) \rangle$.) -5 for strange arguments.

Section 10, Problem 35

You have to show that the map $gH \mapsto Hg^{-1}$ is well-defined and gives a bijection between the set of left cosets and the right cosets of H. 2 points for each. (well-definedness, injective, and surjective). For the well-definedness, you have to show that $g_1 H = g_2 H$ implies $Hg_1^{-1} = Hg_2^{-1}$. If you mentioned that $Hg^{-1} = (gH)^{-1} = \{(gh)^{-1} : h \in H\}$, this implies well-definedness.

There are some students who suggest wrong bijections which don't work. For example, the map $gH \mapsto Hg$ is not well-defined, unless H is a normal subgroup of G. For example, let $G = S_3$, $H = \langle (1\,2) \rangle$, $g_1 = (1\,2\,3)$, $g_2 = (1\,2\,3)(1\,2) =$

$(1\,3)$. Then $g_1H = g_2H$ but $Hg_1 = \{(1\,2\,3), (2\,3)\}$ and $Hg_2 = \{(1\,3), (3\,1\,2)\}$. For such maps, I gave no points.

Also, some students showed that the number of elements in gH is same as the number of elements in Hg for all $g \in G$, but this is not we want in the problem. No points again for such solutions.

Section 11, Problem 6

-1 or -2 for minor mistakes, -5 for big mistakes.