Math 113(4) - Comments for HW13

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Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use LaTeX.
4. For questions that requires proofs, I almost not give any partial credits.

Problem 1

(a) 2 points
(b) 1 point.
(c) Show that $R^\times$ is cyclic (1 point) and R is a field (1 point).

Problem 2

2.5 points for each directions.

Problem 3

You can directly show that the smallest principal ideal containing $(2, x)$ is (1) (by showing that $(2, x)$ is a maximal ideal). You can also show that the only element in $\mathbb{Z}[x]$ that divide both 2 and x is ± 1. This is possible because $\mathbb{Z}[x]$ is UFD (otherwise, you can do in this way).

Problem 4 (Section 45, Exercise 25)

You have to show that if $p = ab$, then at least one of a, b is a unit.

Problem 5 (Section 45, Exercise 26)

You have to use factorization of an element as irreducible elements. What you have to show is that p is a *prime*: if $p|ab$, then $p|a$ or $p|b$. You can't assume that $p = ab$. (Some of you assume $p = ab$ then show $p|a$ or $p|b$, which doesn't show that p is a prime)