Math 113(4) - Comments for HW2

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Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use LaTeX.
4. For questions that requires proofs, I almost not give any partial credits.

Problem 1

2.5 points for each problem.

Problem 2

This problem is wrong - $\mathbb{C}$ and H aren't group with a multiplication, because of the existence of $0 \in \mathbb{C}$ and $O \in H$. However, if we exclude zeros from them, it become a group. So the correct statement of (b) should be that $(\mathbb{C}\backslash\{0\}, \cdot)$ and $(H\backslash\{O\}, \cdot)$ are isomorphic. (Actually, both $\mathbb{C}$ and H are *rings*, which have both addition and multiplication). Anyway, you have to show the followings:

- ϕ is injective and surjective, hence bijective. (This is trivial, but at least you have to mention it. 1 point.)
- For (a), ϕ is an additive homomorphism: $\phi(z+w) = \phi(zw)$. (2 points)
- For (b), ϕ is a multiplicative homomorphism: $\phi(zw) = \phi(z)\phi(w)$. (2 points)

Problem 3

- For a), you have to show that if $a, b \neq 1$, then $a * b \neq 1$. 1 point.
- For b), you have to check that $*$ is associative, 0 is an identity and $-a/(a+1)$ is an inverse of $a \in S$. (1 point for each)
- For c), I give 0.5 point if there's a mistake with calculation.

Problem 4

There are some strange arguments that pretend to use Pigeonhole principle, and I gave partial credits for them. If you didn't use finiteness of G at all, then it should be wrong.

Problem 5

- For c), you have to check that $\phi(G)$ is nonempty, closed under the operation, has an identity (which is $\phi(e)$), and inverse is well-defined. 1 point each.