Math 113(4) - Comments for HW3

Seewoo Lee

September 16, 2018

Some general comments:

1. **DO NOT CHEAT!**
2. Please use staplers or clips, not just fold the left-upper corner of papers!
3. Try to write well! - maybe this will be harder than the first one...
4. If you can, try to use LaTeX.
5. For questions that requires proofs, I almost not give any partial credits.

Problem 1 (Section 5, Problem 26)

-1 point for each wrong answer. If you didn't mention every generator, then -0.5 points each.

I only saw the answer when I graded it, but you have to know the reasons for each problems. For example, $(\mathbb{Q}, +)$ is not cyclic since if $p/q \in \mathbb{Q}$ is a generator of the group, where p and q are coprime integers, then we should have $n(p/q) = 1/2q$ for some $n \in \mathbb{Z}$, which is equivalent to $2np = 1$, and this is impossible.

For G_6, assume that it is cyclic. Then if $a + b\sqrt{2}$ is a generator, then there exists $m, n \in \mathbb{Z}$ s.t. $m(a + b\sqrt{2}) = 1$ and $n(a + b\sqrt{2}) = \sqrt{2}$, which implies $n/m = \sqrt{2}$, contradicts to the irrationality of $\sqrt{2}$.

Problem 2 (Section 5, Problem 45)

The *only if* part, which is fairly easy, is 1 points and *if* part is 4 points. You should show the both directions. For the *if* part, you have to show that

1. There exists an identity (1 point),
2. every elements in H has an inverse (1.5 points),
3. and H is closed under the operation (1.5 points).

It is really important to show in the above order. For example, you can't say anything about the existence of the inverse before you show the existence of the identity element. I deducted some points if your order is different from the above. Also, when you show the existence of the identity element, you have to use nonemptyness of H to *choose* an element a in H. However, there are very few people who mentioned about this, so I didn't deduct any point for this.

Problem 3 (Section 5, Problem 47)

You have to show that

1. $e \in H$ (1 point)
2. Each element in H has inverse (2 points)
3. H is closed under the operation (2 points)

Problem 4 (Section 5, Problem 54)

You have to show that

1. $e \in H \cap K$ (1 point)
2. Each element in $H \cap K$ has inverse (2 points)
3. $H \cap K$ is closed under the operation (2 points)

Problem 5 (Section 6, Problem 56)

I don't know why, but there are several people who solve different problems (Let G be a group and let $G_n = \{x^n : x \in G\}$. Under what hypothesis about G can we show that G_n is a subgroup of G?). Please check the edition of the book you using now. I can' give any points for those people.

Anyway, you have to show that

1. For a) (3 points), you may show that ab is a generator of the group, where $H = \langle a \rangle$ and $K = \langle b \rangle$. If you suggest a right candidate but argument is wrong, then you can get only 1 point. For example, the following is the **wrong argument**: assume that $(ab)^n$ for some $m \geq 1$. Then $a^m b^m = e$, so we should have $r|m$ and $s|m$, which implies $rs|m$. Such argument doesn't work for b).
2. For b) (2 points), you can't take ab as a generator of an order $\text{lcm}(r, s)$ subgroup. For example, let $G = H = K = \mathbb{Z}/4\mathbb{Z}$ and let $a = b = \overline{1}$. Then $ab = \overline{2}$ has order 2, not $4 = \text{lcm}(4, 4)$. You may use a) in the proof, and it you use it properly, I gave full credit even if you didn't get 3 points in a).

Problem 6

You must mention all possible generators for any given group. For example, the subgroup $\{\overline{0}, \overline{3}, \overline{6}, \overline{9}\}$ can be generated by $\overline{3}$ and $\overline{9}$. Also, you have to draw the line that indicate inclusions properly. If A is a proper subgroup of B and B is a proper subgroup of C, then you don't need to draw a line between A and C. But you should draw a line if A is a proper subgroup of B and there's no other subgroup between A and B. -0.5 points for each missing line, and -0.5 points if you didn't give a complete list of generators. -1 point for each missing subgroup. Also, If A is a proper subgroup of B, B should be on the above of A, not below A.