Math 113(4) - Comments for HW2

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Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use LaTeX.
4. For questions that requires proofs, I almost not give any partial credits.

Problem 1

This problem show that the center of D_n is $\{1, r^k\}$.

- To show that $z = r^k$ is in the center, you may have to check that $gr^k = r^k g$ for all $g \in D_n$. Since every elements of D_n is of a form of r^i or sr^i, you only need to check for these cases. (1+1 points)
- To show that r^k is the only nontrivial element in the center, you may assume that $x = r^i$ and $y = sr^j$ are in the center and show that we must have $i = k$. For x, $sx = xs$ gives $i = k$. For y, $ry = yr$ gives $r^2 = e$, which is impossible since $n \geq 4$. It is not sufficient to show that $sy = ys$ implies $j = k$, since you still need to show that sr^k can't be in the center. (1.5+1.5 points)

If you only showed that $sy = ys$ implies $j = k$ in part 2, I only gave 0.5 points for this.

Problem 2

The first one is not cyclic, since it is not abelian. The second one is cyclic which is generated by r or r^5. For the first one, you should mention why it is not a cyclic group. (If not, -0.5 points).You can't just say that it can't be generated by a single element. Similarly, you must give a generator of the second one. (If not, -0.5 points). -0.5 points for small mistakes.

Problem 3

- For i), -0.5 points for each wrong answer.
- For ii), -0.5 points for each wrong answer.
- For iii), 1 point. Note that *order of* $\sigma\tau$ *is least common multiple of* $|\sigma|$ *and* $|\tau|$ can't be an answer because its FALSE. Consider $\sigma = (123)$ and $\tau = (12)$.

Problem 4 (Section 8, Problem 52)

You have to show that

- ρ_a is a bijection from G to G. (1 point)
- The map $\phi : G \to S_G$ defined as $a \mapsto \rho_{a^{-1}}$ is a group homomorphism (2 points), which is one-to-one (2 points).

Note that the map $\phi : a \mapsto \rho_a$ is NOT a group homomorphism, since it satisfies $\phi(ab) = \phi(b)\phi(a)$, instead of $\phi(ab) = \phi(a)\phi(b)$.

Problem 5 (Section 9, Problem 29)

If every elements of H are even, we are done. If it isn't, there exists an odd permutation $a \in H$ and the map $H \to H, x \mapsto ax$ gives a bijection between the set even permutations and odd permutations in H. (You have to show this, otherwise -1 point). Note that you can't take $a = (12)$, since H might not contain a transposition. (Consider $H = \langle (12)(34)(56) \rangle$.)