Math 113(4) - Comments for HW6

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Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use LaTeX.
4. For questions that requires proofs, I almost not give any partial credits.

Problem 1

- For a), you only need to give a right answer, and I gave full credits for them. Here are some common wrong answers for this question:
  - Not all $\phi_a : \mathbb{Z} \to \mathbb{Z}/12\mathbb{Z}$, $n \mapsto an$ are surjective homomorphisms. 1 point.
  - There are 4 such homomorphisms which are determined by $\phi(1) = 1, 5, 7, 11$, not only $a \mapsto a \,(\mathrm{mod} 12)$. Also 1 point.
- For b), you have to prove that the only such homomorphism is trivial one.

Problem 2

No point for strange arguments.

Problem 3

You have to check both directions (3+2 points). For the first part (G is infinite $\Rightarrow$ G has infinitely many distinct subgroups), It is not true that $x \neq y$ implies $\langle x \rangle \neq \langle y \rangle$. If you use such argument in your proof, you can only get 1 point.

For the opposite direction, the most simplest way is to prove contraposition: if G is a finite group, then there exists finitely many *subsets*, so finitely many subgroups. Sum of you just mentioned that if G has infinitely many distinct subgroups, then we can choose distinct elements from each subgroups, and I think this is not trivial at all. In fact, this is not true for *subsets* - for any infinite set A, we can't find an injective function from 2^A to A. (This means that you can't prove the above statement with only set theoretic arguments, unless you try to prove contraposition). So you should prove your claim in a right way, otherwise you can only get 1 point for this direction.

Problem 4

2.5 points for each direction.