Math 113(4) - Comments for HW8

Seewoo Lee

October 20, 2018

Some general comments:

1. Please use staplers or clips, not just fold the left-upper corner of papers!
2. Try to write well! - maybe this will be harder than the first one...
3. If you can, try to use LaTeX.
4. For questions that requires proofs, I almost not give any partial credits.

Problem 1 (Section 14, Exercise 39)

You have to show well-definedness (2.5 points) and it is a homomorphism (2.5 points).

Problem 2 (Section 15, Exercise 40)

1. HN is a subgroup of G:
   (a) HN is nonempty and has an identity. (+0.5)
   (b) HN is closed under multiplication. (+1.5)
   (c) HN is closed under taking inverse. (+1.5)
2. HN is a smallest subgroup of G containing both N and H. (+1.5)

Note that for $n \in N$ and $h \in H$, $nh \neq hn$ in general.

Problem 3

To use first isomorphism theorem you must check all the conditions.

1. For a),
   (a) Define $\phi : \mathbb{C}^\times \to U$ as $z \mapsto z/|z|$, or $re^{i\theta} \mapsto e^{i\theta}$ in a polar coordinate. (+1)

(b) ϕ is a homomorphism. (+0.5)
(c) ϕ is surjective. (+0.5)
(d) $\ker(\phi) = \mathbb{R}^+$. (+0.5)

2. For b),

   (a) Define $\phi : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ as $(a, b) \mapsto 2a - b$. (+1)
   (b) ϕ is a homomorphism . (+0.5)
   (c) ϕ is surjective. (+0.5)
   (d) $\ker(\phi) = \langle (1, 2) \rangle$. (+0.5)

Without first isomorphism theorem, you can define an appropriate homomorphism from $\mathbb{C}^\times/\mathbb{R}^+ \to U$ and show its well-definedness, homomorphism, injectivity and surjectivity.

Problem 4

For a), unit of $\mathbb{Z}$ is 1 and -1. If you only mentioned 1, I deducted 0.5 point. Also, $0 \in \mathbb{Q}$ is not a unit in $\mathbb{Q}$, and if you contained $(1, 0, 1)$ in your answer, I deducted 0.5 point.
For b), you have to show that if such homomorphism exists, then it should be $\phi(2) = 3$ or $\phi(2) = -3$ (1.5 points), and this gives a contradiction since $\phi(4) = \phi(2+2) = \phi(2)+\phi(2) = 6$ but $\phi(4) = \phi(2\times 2) = \phi(2)\times\phi(2) = 9$ (1 point). You can also show in the following way: if $\phi(2) = 3n$, then $\phi(2 \times 2) = \phi(2 + 2)$ implies $9n^2 = 6n$, which gives $n = 0$, so ϕ is not injective.

Problem 5 (Section 18, Exercise 50)

You have to show that I_a is

1. closed under addition, (+1.5)
2. closed under multiplication, (+1.5)
3. closed under additive inverses, (+1.5)
4. and nonempty. (+0.5)