Math 113(4) - Comments for HW9

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Some general comments:

- 1. When you refer a theorem in the textbook, try to say what the theorem is about, not just saying that by the Theorem 15.xx.
- 2. Please use staplers or clips, not just fold the left-upper corner of papers!
- 3. Try to write well! maybe this will be harder than the first one...
- 4. If you can, try to use LATEX.
- 5. For questions that requires proofs, I almost not give any partial credits.

Problem 1 (Section 19, Exercise 29)

4 points for proofs that need more explanation. 3 points for proofs that hard to understand what they are saying. Here are some comments:

- Characteristic of a ring is a natural number (positive integer or 0), not an element of a given ring.
- Try to define first that you are going to use, not just using it first.

Problem 2 (Section 21, Exercise 2)

If you only show that $Q_D \subseteq \mathbb{Q}(\sqrt{2})$, you can get only 4 points. You have to show the other direction (which is very easy to check). There are many students who only checked one direction by using the rationalization of a denominator.

Problem 3

- 1. $\mathbb{Z}[i]$ is a subgroup under addition. (1 point)
- 2. $\mathbb{Z}[i]$ is closed under multiplication. (1 point)
- 3. $\mathbb{Z}[i]$ is not a field. (1 point)

- 4. $\mathbb{Z}[i]$ is an integral domain.
 - (a) $\mathbb{Z}[i]$ contains a unity 1. (0.5 points)
 - (b) $\mathbb{Z}[i]$ is commutative. (0.5 points)
 - (c) $\mathbb{Z}[i]$ has no zero divisor. (1 point)

Problem 4

There are a lot of students who abuse notations, which make you confuse and lead to wrong answers. For $a \in \mathbb{Z}$, \overline{a} usually denotes its image in $\mathbb{Z}/n\mathbb{Z}$, but some of students use \overline{a} for both element in \mathbb{Z} and $\mathbb{Z}/n\mathbb{Z}$. Also, for both (a) and (b), you have to show both if and only if direction. If you show only one, you can only get 1.5 points.