

Quiz 1

True/False - No explanation needed. (2pts)

1. If a set S is infinite, then the number of subset of S , $|\mathcal{P}(S)|$, is also infinite. **True/False**

sol. For each element $a \in S$, we have a subset $\{a\} \in \mathcal{P}(S)$. Thus each element in S gives a subset of S which are distinct, so there are infinite number of subsets of S .

2. To find how many positive integers $\leq n$ are divisible by d we calculate the fraction $\frac{n}{d}$ and round down in order not to miss any numbers. **True/False**

sol. What I intended is True: From 1 to n , there are $\lfloor n/d \rfloor$ many numbers that are divisible by d : $d, 2d, 3d, 4d, \dots, \lfloor n/d \rfloor d$. However, it may be false because the reason we round down is in order not to *overcount*. **So I give everyone 1 point for this problem.**

Problems - Need justification. No justification means **zero!**

1. How many license plates can be made using either four digits followed by four uppercase English letters or four uppercase English letters followed by four digits? (5pts)

sol. There are two sets of license plates to consider: Case 1: three digits followed by three uppercase letters. There are 10 ways to pick each of the digits, and 26 ways to pick each of the letters: $10^4 \times 26^4$ Case 2: three uppercase letters followed by three digits. There 26 ways to pick each of the letters, and 10 ways to pick each of the digits: $26^4 \times 10^4$. So the answer is $2 \times 26^4 \times 10^4$.

2. How many 4-digits numbers that contain at least one 0? (5 points)

sol. There are $9 \times 10 \times 10 \times 10 = 9000$ 4-digits numbers. Among them, there are $9 \times 9 \times 9 \times 9 = 6561$ 4-digits numbers that do not contain any 0. By the subtraction rule, we have $9000 - 6561 = 2439$ 4-digits numbers that contain at least one 0.

Note. 4-digit numbers and strings of 4 decimal digits are different. First one can't start with 0, but the latter one can. For example, we don't count 0123 as a 4-digit number, but we count it as a string of 4 decimal digits. I gave partial credits to someone who consider it as latter one.