MATH 10B with Prof. Stankova
DIS 102; TTh 8:00~ 9:30am
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Quiz 10

True/False - No explanation needed. (2pts)

1. The formula for the standard deviation of a continuous RV is a limit version of the standard deviation for a discrete RV, and both always exist. True/**False**

   sol. Standard deviation of a continuous RV may not exists.

2. Chebyshev's inequality is useful only when $k > 1$. **True**/False

   sol. When $k \leq 1$, Chebyshev's inequality only gives $P(|X - \mu| \geq k\sigma) \leq 1 - \frac{1}{k^2}$, which is not useful since $1 - \frac{1}{k^2} \leq 0$ and probability is always nonnegative.

Problems - Need justification. No justification means **zero**!
Let X be a continuous random variable with a PDF

$$f(x) = \begin{cases} \frac{x}{2} & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

1. Compute the standard error σ of X. (5pts)

   sol. First, the expected value $\mu = E[X]$ is

   $$\int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{x^2}{2}dx = \left[\frac{x^3}{6}\right]_0^2 = \frac{4}{3}.$$

   Then the variance $\sigma^2 = \text{Var[X]}$ is

   $$\sigma^2 = E[X^2] - \mu^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \frac{16}{9} = \int_0^2 \frac{x^3}{2}dx - \frac{16}{9} = \left[\frac{x^4}{8}\right]_0^2 - \frac{16}{9} = 2 - \frac{16}{9} = \frac{2}{9}.$$

   So $\sigma = \sqrt{\text{Var}[X]} = \frac{\sqrt{2}}{3}$.

2. Estimate the probability

   $$P\left(\frac{2}{3} \leq X \leq 2\right)$$

   using the Chebyshev's inequality. (Find a lower bound) (5pts)

   sol. The inequality can be written as

   $$\frac{2}{3} \leq X \leq 2 \Leftrightarrow -\frac{2}{3} \leq X - \frac{4}{3} \leq \frac{2}{3} \Leftrightarrow \left|X - \frac{4}{3}\right| \leq \frac{2}{3} = \sqrt{2} \cdot \frac{\sqrt{2}}{3}$$

   which has a form of $|X - \mu| \leq k\sigma$ where $k = \sqrt{2}$. By Chevyshev's inequality, we get

   $$P\left(\frac{2}{3} \leq X \leq 2\right) = P(|X - \mu| \leq \sqrt{2}\sigma) \geq 1 - \frac{1}{(\sqrt{2})^2} = \frac{1}{2}.$$