MATH 10B with Prof. Stankova
DIS 106; TTh 9:30~11:00
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Tue 4/9/19

Quiz 10

True/False - No explanation needed. (2pts)

1. For a symmetric distribution centered at 0, we do not have to calculate σ because it will always be 0 or not well-defined. True/**False**

   sol. For a symmetric distribution centered at 0, μ and m always be 0 or not well-defined, but not σ. Think about the standard normal distribution.

2. Chebyshev's inequality guarantees that 75% of the sample data for any probability distribution lies within 2 standard deviations of its mean. **True**/False

   sol. We have $P(|X-\mu| \leq 2\sigma) \geq 1 - \frac{1}{2^2} = \frac{3}{4} = 0.75$.

Problems - Need justification. No justification means **zero**!

1. Let X be a geometric random variable with $p = \frac{1}{4}$. (Assume that X counts the number of failure until success.) Using Chebyshev's inequality, estimate the probability $P(X \leq 9)$. (Find a lower bound.) You can use

   $$P(X \leq 9) = P(-3 \leq X \leq 9).$$

   Also, compute the exact value. (10pts)

   sol. The mean and the standard errors are given by

   $$\mu = E[X] = \frac{1-p}{p} = 3, \qquad \sigma = \sqrt{\mathrm{Var}[X]} = \sqrt{\frac{1-p}{p^2}} = 2\sqrt{3}.$$

   The inequality appears in the probability can be written as

   $$-3 \leq X \leq 9 \Leftrightarrow -6 \leq X - 3 \leq 6 \Leftrightarrow |X-3| \leq 6 = \sqrt{3} \cdot 2\sqrt{3}$$

   which has a form of $|X-\mu| \leq k\sigma$ where $k = \sqrt{3}$. By Chebyshev's inequality, we get

   $$P(X \leq 9) = P(-3 \leq X \leq 9) = P(|X-\mu| \leq \sqrt{3}\sigma) \geq 1 - \frac{1}{(\sqrt{3})^2} = \frac{2}{3}.$$

   For the exact value, we have

   $$P(X \leq 9) = 1 - P(X \geq 10) = 1 - [(1-p)^{10}p + (1-p)^{11}p + \cdots] = 1 - (1-p)^{10} = 1 - \left(\frac{3}{4}\right)^{10}.$$