

True/False - No explanation needed. (2pts)

1. A normal distribution is completely determined by its μ and σ . **True/False**
2. When using a random variable X and one experiment with it to estimate a parameter θ , we compare all values of $L(x|\theta)$ for $X = x$ fixed and θ varying. **True/False**

Problems - Need justification. No justification means **zero!**

Prof. Stankova counted number of students who attended the lecture everyday, and she found that there were 400 students who came to the class yesterday. Let λ be an average number of students who came to the class.

Specify the distribution that the number of students may follows, and find the maximum likelihood function fo the given data. Also, find the maximum likelihood estimate for λ given this data. You can use derivative or log-derivative. (10pts)

sol. X follows Poisson distribution with the parameter λ . **What I intended is Poisson distribution, but one may think this as binomial distribution where the total number of students is given. Here we will assume that there can be arbitrary many students, and X follows Poisson distribution.** Then the maximum likelihood function is given by

$$L(\lambda|400) = P(X = 400|\lambda) = \frac{\lambda^{400} e^{-\lambda}}{400!}.$$

To find the maximum likelihood estimate, we have to find λ that maximizes the function. By taking log-derivative, we have

$$\begin{aligned}\log L(\lambda|400) &= 400 \log \lambda - \lambda - \log(400!) \\ \frac{d}{d\lambda} L(\lambda|400) &= \frac{400}{\lambda} - 1 = 0\end{aligned}$$

By solving the equation, we get $\hat{\lambda} = 400$.