MATH 10B with Prof. Stankova
DIS 106; TTh 9:30∼11:00
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Quiz 11

Student:
SID:
Tue 4/16/19

True/False - No explanation needed. (2pts)

1. If we want to find a 99% confidence interval, we need to change the constant 2 appears in the formula of 95% interval to a bigger number. **True**/False

   sol. The constant 2 in the formula corresponds to a z-score that gives $2 \cdot z(2.0) \approx 0.95$. For 99% or higher confidence, we have to adjust this z-score with bigger one so that $2 \cdot z(t_0) \approx 0.99$. (For example, we use $t_0 = 2.56$.)

2. The Maximum Likelihood (M-L) method uses a probabilistic experiment to estimate a real world parameter θ by considering all possible outcomes of the experiment. True/**False**

   sol. The M-L does not necessarily consider all possible outcomes of an experiment, rather it may consider only one possible outcome or observation of data and then make a likelihood estimation of θ based on that observation and the associated joint probability.

Problems - Need justification. No justification means **zero**!

Assume that we are flipping a biased coin until we get a head. We did this process 10 times and we counted the number of tails until we get a head:

$$6, 7, 11, 9, 13, 8, 10, 9, 12, 5.$$

Let p be a probability that head comes out when we flip the coin once.

Estimate the probability p and the variance σ^2. For the variance, compute it **by using two different estimators** - universal one or the specific one for specific distribution. You can assume that the sum of above numbers is 90 and the sum of squares is 870.

Also, compute the 95% confidence interval (you can use any of the estimators above). (10pts)

sol. The average of geometric distribution is $\mu = E[X] = \frac{1-p}{p} = \frac{1}{p} - 1$. Then $p = \frac{1}{1+\mu}$, so we can estimate p by using the estimator of μ:

$$\hat{\mu} = \frac{6+7+11+9+13+8+10+9+12+5}{10} = \frac{90}{10} = 9, \quad \hat{p} = \frac{1}{1+\hat{\mu}} = \frac{1}{10}.$$

For variance, we have two estimators that we can use.

1. (Only for geometric distribution) The variance of geometric distribution is given by $\sigma^2 = \frac{1-p}{p^2}$. So we can estimate it as

$$\hat{\sigma}^2 = \frac{1-\hat{p}}{\hat{p}^2} = 90.$$

2. (Universal estimator) We have another estimator - universal one.

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2.$$

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We can write the sum of squares as

$$\sum_{i=1}^{n}(x_i - \overline{x})^2 = \sum_{i=1}^{n}(x_i^2 - 2x_i\overline{x} + \overline{x}^2) = \sum_{i=1}^{n} x_i^2 - n\overline{x}^2$$

so by using the given data, we get

$$\hat{\sigma}^2 = \frac{1}{9}(870 - 10 \cdot 9^2) = \frac{60}{9} = \frac{20}{3}.$$

(You can see that there's a quite big gap between two estimators). The 95% confidence interval (for μ) is given by

$$\left(9 - 2\frac{\sqrt{90}}{\sqrt{10}}, 9 + 2\frac{\sqrt{90}}{\sqrt{10}}\right) = (3, 15)$$

or

$$\left(9 - 2\sqrt{\frac{20}{30}}, 9 + 2\sqrt{\frac{20}{30}}\right) \approx (7.37, 10.63).$$