

Quiz 12

True/False - No explanation needed. (2pts)

1. Z -statistic is appropriate for small sample size. **True/False**

sol. Z -statistic is appropriate for large sample size, so that we can approximate it as standard normal distribution (by using the Central Limit Theorem).

2. The higher the significance α of a test, the higher the probability of rejecting a true null hypothesis. **True/False**

sol. The significance α measures the probability of making Type 1 error (rejecting a true null hypothesis).

Problems - Need justification. No justification means **zero!**

Boys of a certain age are known to have a mean weight of $\mu = 85$ pounds. A complaint is made that the boys living in a municipal children's home are *underfed*. As one bit of evidence, $n = 100$ boys (of the same age) are weighed and found to have a mean weight of $\bar{x} = 80$ pounds. It is known that the population standard deviation σ is 20 pounds.

1. Assume that we want to determine whether the average is 85 or smaller. Set the appropriate null hypothesis H_0 and alternative hypothesis H_1 . Is this one-sided or two-sided? (4pts)

sol.

$$\begin{cases} H_0 : \mu = 85 \\ H_1 : \mu < 85 \end{cases}$$

This is one-sided.

2. By using Z -statistic, draw a conclusion (reject H_0 or not?). Use the significance level $\alpha = 0.02$. You can use the following standard normal table. (5pts)

z	0.00	0.01	0.02	0.03	0.04
1.9	0.4713	0.4719	0.4726	0.4732	0.4738
2.0	0.4772	0.4778	0.4783	0.4788	0.4793
2.1	0.4821	0.4826	0.4830	0.4834	0.4838
2.2	0.4861	0.4864	0.4868	0.4871	0.4875
2.3	0.4893	0.4896	0.4898	0.4901	0.4904
2.4	0.4918	0.4920	0.4922	0.4925	0.4927
2.5	0.4938	0.4940	0.4941	0.4943	0.4945

sol. Assume that the null hypothesis is true. Then the Z -score is

$$Z = \frac{X - \mu}{\sigma/\sqrt{n}} = \frac{80 - 85}{20/\sqrt{100}} = -2.5,$$

so the p -value is

$$P(X \leq 80) = P(Z \leq -2.5) = 0.5 - z(2.5) = 0.5 - 0.4938 = 0.0062 < \alpha = 0.02,$$

so we reject H_0 .

3. What can we do if the population standard deviation is unknown and the sample size is small? (1pt)

sol. We can use T -statistic instead of Z -statistic.