MATH 10B with Prof. Stankova
DIS 106; TTh 9:30∼11:00
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Quiz 12

Student:
SID:
Tue 4/23/19

True/False - No explanation needed. (2pts)

1. The alternative hypothesis is a theory that we believe is false. True/**False**

   sol. The alternative hypothesis is just an hypothesis that is different from the null hypothesis - no matter it is what we believe or not.

2. Maximum likelihood estimator of mean for normal distribution is unbiased. **True**/False

Problems - Need justification. No justification means **zero**!

You suspect that a 12-sided die is biased toward coming up 12s. (The possible rolls are from 1 to 12). Let $H_0 : p = \frac{1}{12}$ and $H_1 : p > \frac{1}{12}$, where p is the probability that 12 comes out.

1. If you get two 12 in three rolls, what can you conclude with significance level $\alpha = 0.05$? (5pts)

   sol. Let X be the number of 12s, so that it follows the binomial distribution with $n = 3$ and $p = 1/12$. The p-value is

   $$P(X \geq 2) = \binom{3}{2}\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right) + \binom{3}{3}\left(\frac{1}{12}\right)^3 = \frac{34}{1728} < \alpha = \frac{1}{20} = 0.05,$$

   so we reject H_0.

2. If you get twenty-two 12 in 176 rolls, what can you conclude with significance level $\alpha = 0.05$? You may use Z-test with the following standard normal table. (Note that $176 = 11 \cdot 16$.) (5pts)

z	0.00	0.01	0.02	0.03	0.04
1.9	0.4713	0.4719	0.4726	0.4732	0.4738
2.0	0.4772	0.4778	0.4783	0.4788	0.4793
2.1	0.4821	0.4826	0.4830	0.4834	0.4838
2.2	0.4861	0.4864	0.4868	0.4871	0.4875
2.3	0.4893	0.4896	0.4898	0.4901	0.4904
2.4	0.4918	0.4920	0.4922	0.4925	0.4927
2.5	0.4938	0.4940	0.4941	0.4943	0.4945

   sol. The Z-statistic is

   $$z = \frac{(\bar{x} - p)\sqrt{n}}{\sqrt{p(1-p)}} = \frac{\left(\frac{22}{176} - \frac{1}{12}\right)\sqrt{176}}{\sqrt{\frac{1}{12}\frac{11}{12}}} = \frac{\frac{1}{24} \cdot 4\sqrt{11}}{\frac{\sqrt{11}}{12}} = 2$$

and the p-value is

$$P(X \geq 22) \approx P(Z \geq 2) = 0.5 - z(2.0) = 0.5 - 0.4772 = 0.0228 < \alpha = 0.05,$$

so we reject H_0.