MATH 10B with Prof. Stankova
DIS 102; TTh 8:00∼ 9:30am
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Student:
SID:
Tue 2/5/19

Quiz 2

True/False - No explanation needed. (2pts)

1. If $\lfloor x \rfloor \neq \lceil x \rceil$, then x is not an integer. **True**/False

   sol. If x is an integer, then $\lfloor x \rfloor = x = \lceil x \rceil$.

2. For any 9 people in a line, all with different heights, we can find 4 people – without rearranging them and not necessarily consecutive – whose heights are already arranged in an increasing order, or in a decreasing order. True/**False**

   sol. Assume that the 9 people have the following heights: $3, 2, 1, 6, 5, 4, 9, 8, 7$. In this sequence, you can't find any subsequence of length 4 that is increasing or decreasing. (However, if there are 10 people, then there exists such subsequence, as we proved in the lecture.)

Problems - Need justification. No justification means **zero**!

1. How many permutations of the letters $ABCDEFGH$ that A, B are adjacent, and C, D are adjacent? For example, we count strings like $BAECDFGH$. (5pts)

   sol. First, we consider AB as one block and CD as another block. Then we have 6 things to permute: $\{AB, CD, E, F, G, H\}$. So we have $P(6, 6) = 6!$ possibilities. For each permutation, A and B can be swapped and C and D can be swapped, so we have to multiply $2! \times 2!$, and the answer is $6! \times 2! \times 2! = 2880$.

2. Prove that if n, k are integers with $1 \leq k \leq n$, then

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

   Here $\binom{n}{k} = C(n, k)$. (5 points)

   sol. By the definition of binomial coefficient, we have

$$k\binom{n}{k} = k \times \frac{n!}{k!(n-k)!} = \frac{n!}{(k-1)!(n-k)!} = n \times \frac{(n-1)!}{(k-1)!(n-k)!} = n\binom{n-1}{k-1}.$$