MATH 10B with Prof. Stankova
DIS 106; TTh 9:30∼11:00
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Tue 2/5/19

Quiz 2

True/False - No explanation needed. (2pts)

1. $0! = 0$. True/**False**

   sol. $0! = 1$. Be careful! There are some possible justification for this. For example, $C(n, n) = 1$ since there are only one way to choose n elements from n given elements. Now use $C(n, n) = n!/(n! \times (n-n)!) = n!/(n! \times 0!)$.

2. The binomial coefficients first increase from left to right along a row in Pascal's triangle, but then they decrease from the middle to the end of the row. **True**/False

   sol. To *prove* this, we have

$$\frac{C(n, k+1)}{C(n, k)} = \frac{n!}{(k+1)!(n-k-1)!} \times \frac{k!(n-k)!}{n!} = \frac{n-k}{k+1},$$

   so $C(n, k+1) > C(n, k)$ when $n-k > k+1$, which is equivalent to $k < \frac{n-1}{2}$, and $C(n, k+1) \leq C(n, k)$ when $k \geq \frac{n-1}{2}$. Hence $C(n, k)$ increases from the left to the middle, and decreases from the middle to the end.

Problems - Need justification. No justification means **zero**!

1. How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ to guarantee that at least one **pair** of these numbers add up to **7**? (5pts)

   sol. First, divide the set as

$$\underbrace{\{1, 6\}, \{2, 5\}, \{3, 4\}}_{A}, \underbrace{\{7\}, \{8\}, \{9\}, \{10\}}_{B}.$$

   We want to find number that guarantees to select two numbers that belongs to the same pair among the first three pairs (A). First, 7 is not enough, since we can choose $1, 2, 3, 7, 8, 9, 10$, and among these 7 numbers, there are no two numbers that add up to 7.

   To show that 8 is enough, since there are 4 sets with one element each in B, there are at least 4 numbers that belongs to A. Now by PHP (numbers are pigeons and the sets in A are holes, so ≥ 4 pigeons and 3 holes), there exists two numbers belongs to the same set in A, which add up to 7.

2. How many ways are there for 5 women and 5 men to stand in a line so that no two men stand next to each other **and** no two women stand next to each other? (5 points)

   sol. The condition means that they have to stand in alternating way: WMWMWMWMWM or MWMWMWMWMW. For each cases, we have 5! possibilities for permuting women and 5! possibilites for permuting men. Hence there are $2 \times 5! \times 5! = 28800$ ways to do this.