MATH 10B with Prof. Stankova
DIS 102; TTh 8:00∼ 9:30am
GSI: Seewoo Lee

Quiz 3

Student:
SID:
Tue 2/12/19

True/False - No explanation needed. (2pts)

1. The equation $x_1 + x_2 + x_3 + x_4 = 10$ in positive integers has as many solutions as trying to feed 4 (different) dogs with 10 (identical) biscuits. True/**False**

   sol. The number of solutions of the equation in positive integers is same as the number of ways to feed 4 different dogs with 10 identical biscuits *where each dog eat at least one biscuits*, which is $C(10-1, 10-4) = C(9, 6)$. However, the latter one (without any restriction on surjectivity) is $C(10+4-1, 10) = C(13, 10)$.

2. The Quick Sort algorithm is, on the average, faster than the Bubble Sort algorithm because the number of inversions in the list being sorted decreases faster during the Quick Sort algorithm. **True**/False

Problems - Need justification. No justification means **zero**!

1. How many ways there to distribute 8 indistinguishable balls into 4 distinguishable bins if each bin contains at least one ball? (5pts)

   sol. The formula gives $C(8-1, 8-4) = C(7, 4)$. Or, we can first distribute 4 balls into 4 distinguishable bins with 1 ball each, then we have 4 balls left to distribute into 4 distinct bins without any restriction. Hence there are $C(4+4-1, 4) = C(7, 4)$ ways.

2. How many ways are there to deal 10 cards from a deck of 52 cards to 4 players (allowing the possibility of a player getting zero cards) if the players are indistinguishable? (Your answer may be in terms of Stirling numbers.) (5pts)

   sol. First, we choose 10 cards form a deck of 52 cards to distribute, which has $C(52, 10)$ many possibilities. After choosing 10 cards, we have $S(10, 1) + S(10, 2) + S(10, 3) + S(10, 4)$ ways to deal the 10 cards to the 4 players (cards = balls, players = urns). Thus there are $C(52, 10) \times (S(10, 1) + S(10, 2) + S(10, 3) + S(10, 4))$ ways.