MATH 10B with Prof. Stankova
DIS 106; TTh 9:30∼11:00
GSI: Seewoo Lee

Quiz 3

Student:
SID:
Tue 2/12/19

True/False - No explanation needed. (2pts)

1. Some algorithm does not lead to the desired output. **True**/False

   sol. For example, stable matching algorithm doesn't work well for the stable parings of *roommates*.

2. The number of ways to put b **distinct** balls into u **identical** boxes is same as the number of ways to put b **identical** balls into u **identical** boxes, where we only count the cases that each box has at most one ball in it. **True**/False

   sol. Both are given by the same answer

$$\begin{cases} 0 & b > u \\ 1 & b \leq u \end{cases}$$

Problems - Need justification. No justification means **zero**!

1. How many solutions are there to the equation $x_1+x_2+x_3+x_4+x_5 = 20$ where x_1, x_2, x_3, x_4, x_5 are nonnegative integers and $x_1 \geq 4$? (5pts)

   sol. Let $y_1 = x_1 - 4$. Then we have a new equation $y_1 + x_2 + x_3 + x_4 + x_5 = 16$ which is equivalent to the original equation, where y_1, x_2, x_3, x_4, x_5 are nonnegative integers. The number of solutions of the equation is same as the number of ways to put 16 identical balls into 5 distinct bins, which is $C(5 + 16 - 1, 16) = C(5 + 16 - 1, 5 - 1) = C(20, 4)$.

2. How many ways to put 10 distinct balls into 6 distinct bins where each bin has at least one balls? (Your answer may be in terms of Stirling numbers.) (5pts)

   sol. First, assume that all the bins are identical. Then there are $S(10, 6)$ ways to put 10 distinct balls where each bin has at least one balls. Since all the bins are distinct, we have to consider the permutation of 6 bins, so we have to multiply 6! and the answer is $6! \times S(10, 6)$.