MATH 10B with Prof. Stankova
DIS 102; TTh 8:00~ 9:30am
GSI: Seewoo Lee

Quiz 4

Student:
SID:
Tue 2/19/19

True/False - No explanation needed. (2pts)

1. For any events $A, B \subseteq \Omega$, we have $P(A \cup B) + P(A \cap B) = P(A) + P(B)$. **True**/False

   sol. This follows from $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

2. Sending off newly-married couples to honeymoons on different planets around the universe will provide a counterexample for the even version of the "Odd-pie fight" problem. **True**/False

Problems - Need justification. No justification means **zero**!

1. What is the probability that a 5-card poker hand contains no aces? (A 5-card poker hand consists of 5 cards from a 52 card standard deck.) (5pts)

   sol. A probability space Ω for this case is all the possible combinations of 5 cards from 52 cards, which has $C(52, 5)$ elements. Among those, there are 4 aces, so we have 48 cards which are not aces and there are $C(48, 5)$ possible combinations of 5 cards that do not contain any aces. Thus the answer is

$$\frac{C(48,5)}{C(52,5)} = \frac{\frac{48!}{5! \times 43!}}{\frac{52!}{5! \times 47!}} = \frac{47 \times 46 \times 45 \times 44}{52 \times 51 \times 50 \times 49}$$

2. Prove that

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$$

   for all $n \geq 1$, by using mathematical induction. (5pts)

   sol. Basis step: for $n = 1$, we have $1 \cdot 2 = 2 = \frac{1 \cdot 2 \cdot 3}{3}$.

   Inductive step: Assume that the equation is true for some n, so we have

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}.$$

   For $n + 1$,

$$\begin{aligned} & 1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n+1) + (n+1) \cdot (n+2) \\ &= \frac{n(n+1)(n+2)}{3} + (n+1)(n+2) = (n+1)(n+2)\left(\frac{n}{3} + 1\right) \\ &= (n+1)(n+2) \cdot \frac{n+3}{3} = \frac{(n+1)(n+2)(n+3)}{3} \end{aligned}$$

   so it is also true for $n + 1$. Hence, by mathematical induction, the equation is true for all $n \geq 1$.