MATH 10B with Prof. Stankova
DIS 106; TTh 9:30~11:00
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Quiz 4

Student:
SID:
Tue 2/19/19

True/False - No explanation needed. (2pts)

1. Mathematical induction can't be used to prove statements without any explicit mathematical equations. True/**False**

   sol. It can be used to prove statements without any explicit equations. Think about the examples that we did in the class: Odd-pie fighting or dividing squares into smaller squares.

2. When calculating the probability $P(A)$ for some event $A \subseteq \Omega$ on an "equally likely" finite probability space (Ω, P), we can simply count the number of outcomes of A (the good possibilities) and divide that by all outcomes in Ω (all possibilities). **True**/False

Problems - Need justification. No justification means **zero**!

1. What is the probability that when you roll a fair die 8 times, you never get a multiple of 3? (5pts)

   sol. The probability space has 6^8 elements, which are all the possible cases that can occur when we roll a fair dice 8 times. Now there are 4 numbers on a dice which are not multiples of 3 (1, 2, 4, 5), so there are 4^8 number of cases that contains so multiples of 3 while rolling a dice 8 times. Thus the probability is

$$\frac{4^8}{6^8} = \frac{2^8}{3^8}.$$

2. Let $\{a_n\}_{n\geq 1}$ be a sequence defined by $a_1 = 1, a_2 = 4$ and $a_{n+2} = 2a_{n+1} - a_n + 2$. Prove that $a_n = n^2$ for all $n \geq 1$, by using mathematical induction. (5pts)

   sol. Basis step: for $n = 1, 2$, we have $a_1 = 1 = 1^2$ and $a_2 = 4 = 2^2$.

   Inductive step: assume that the statement is true for some n and $n + 1$. So we have $a_n = n^2$ and $a_{n+1} = (n + 1)^2$. Then we have

$$\begin{aligned} a_{n+2} &= 2a_{n+1} - a_n + 2 \\ &= 2(n+1)^2 - n^2 + 2 \\ &= 2n^2 + 4n + 2 - n^2 + 2 \\ &= n^2 + 4n + 4 = (n+2)^2, \end{aligned}$$

   so the equation is also true for $n + 2$. Thus the equation is true for all $n \geq 1$.