MATH 10B with Prof. Stankova
DIS 102; TTh 8:00∼ 9:30am
GSI: Seewoo Lee

Student:
SID:
Tue 2/26/19

Quiz 5

True/False - No explanation needed. (2pts)

1. If $P(A|B) = P(B|A)$, then $P(A) = P(B)$. True/**False**

   sol. If $P(A \cap B) = 0$ and $P(A) \neq P(B)$, then it is still true that $P(A|B) = P(B|A)$ since both are *zero*, but $P(A)$ and $P(B)$ are distinct.

2. Among other things, the proof of Bayes' Theorem for finding $P(B|A)$ depends on being able to split the probability $P(A)$ as sum of probabilities $P(A \cap B)$ and $P(A \cap \overline{B})$, and then further rewrite these as products of certain other probabilities. **True**/False

Problems - Need justification. No justification means **zero**!

1. Suppose you flip a coin three time. What is the conditional probability that exactly two flips are tails, given that at least one of flips is a tail? (5pts)

   sol. Let A be the event that exactly two flips are tails, and let B be the event that at least one of flips is a tail. Since $A \subseteq B$, we have $A \cap B = A$ and $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)}$. Let Ω be the probability space of flipping coins three times, which has $2^3 = 8$ elements. Among those, there are 3 cases that exactly two flips are tails (HTT, THT, TTH), so $P(A) = \frac{3}{8}$. Also, the number of cases with at lease one tail is $2^3 - 1 = 7$ by subtraction rule, so $P(B) = \frac{7}{8}$. By combining these we get $P(A|B) = \frac{3}{7}$.

2. A restaurant has 3 chefs. Suppose that that if a patron eats a meal prepared by Chef A, B, or C, the probability of dissatisfaction is 0.01, 0.02, and 0.03, respectively. Suppose that Chef A makes 50% of the meals, B makes 25%, and C makes 25% of the meals. If a meal was a failure, what is the probability that it was prepared by Chef A? (5pts)

   sol. Let F be an event for failure and A, B, C be events that the food is from A, B, and C, respectively. Then we want to compute $P(A|F)$ with given information

$$P(A) = 0.5, \quad P(B) = 0.25, \quad P(C) = 0.25$$
$$P(F|A) = 0.01, \quad P(F|B) = 0.02, \quad P(F|C) = 0.03.$$

   By Bayes' theorem, we get

$$\begin{aligned} P(A|F) &= \frac{P(F|A)P(A}{P(F|A)P(A) + P(F|B)P(B) + P(F|C)P(C)} \\ &= \frac{0.01 \cdot 0.5}{0.01 \cdot 0.5 + 0.02 \cdot 0.25 + 0.03 \cdot 0.25} = \frac{1}{6} \end{aligned}$$