MATH 10B with Prof. Stankova
DIS 102; TTh 8:00∼ 9:30am
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Tue 3/5/19

# Quiz 6

**True/False** - No explanation needed. (2pts)

1. If two events $A$ and $B$ are disjoint, then they are independent. True/**False**

   *sol.* There's no relation between disjointness and independence. For example, if $A \cap B = \emptyset$, then $P(A \cap B) = 0$. But this doesn't imply $P(A)P(B) = P(A \cap B) = 0$.

2. If the range of a random variable $X$ is a set $T \subset \mathbb{R}$, then we should have $\sum_{t \in T} P(X^{-1}(t)) = 1$. **True**/False

   *sol.* For each $t \in T$, $X^{-1}(t) \subseteq \Omega$ represents disjoint events where the union is the whole outcome space $\Omega$. Hence the sum of probabilities is 1.

**Problems** - Need justification. No justification means **zero**!

1. Two dies were rolled. Are the events that the first die rolled is a 2 and that the two rolls are the same independent? (5pts)

   *sol.* Note that the situation is ELOP. Let $A$ be the event that first die rolled 2, and $B$ the event that the two rolls are the same. The outcome space is

   $$\Omega = \{(1,1), (1,2), \dots, (6,6)\}$$

   and has the size $|\Omega| = 6^2 = 36$. We have $A = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\}$ and $|A| = 6$, so $P(A) = |A|/|\Omega| = 6/36 = 1/6$. Also, we have $B = \{(1,1), (2,2), \dots, (6,6)\}$ and $|B| = 6$, so $P(B) = |B|/|\Omega| = 6/36 = 1/6$. By the way, the intersection has only one possibility $(2,2)$ and $P(A \cap B) = 1/36$. Hence $P(A \cap B) = 1/36 = P(A)P(B)$, which implies that $A$ and $B$ are *independent*.

2. Flip a fair coin 4 times, and let $X$ denote **the product of the number of heads and the number of tails**. Find the probability mass function for $X$. (5pts)

   *sol.* Note that the number of possible outcomes is $2^4 = 16$. First, the range of the random variable $X$ is $\{0, 3, 4\}$, since we have three possibilities: $0 \cdot 4 = 4 \cdot 0, 1 \cdot 3 = 3 \cdot 1, 2 \cdot 2$.

   (a) For $X = 0$, there are only two cases: all heads or all tails. Hence

   $$P(X = 0) = \frac{2}{16} = \frac{1}{8}.$$

   (b) For $X = 3$, each case (3H1T and 1H3T) has $\binom{4}{1} = 4$ possibilities, so we have

   $$P(X = 3) = \frac{8}{16} = \frac{1}{2}.$$

   (c) For $X = 4$, we have $\binom{4}{2} = 6$ cases and

   $$P(X = 4) = \frac{6}{16} = \frac{3}{8}.$$