MATH 10B with Prof. Stankova
DIS 106; TTh 9:30∼11:00
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Quiz 6

Student:
SID:
Tue 3/5/19

True/False - No explanation needed. (2pts)

1. If two events A and B intersects ($A \cap B \neq \emptyset$), then they can't be independent. True/**False**

   sol. Disjointness and independence of events are not related. For example, let's assume that we roll two dies, and let A be the event that the first roll is 1, and let B be the event that the second roll is 1. Then they aren't disjoint but independent.

2. Two show that two random variables are independent, it is enough to find a pair of two ~~events~~ subsets $E, F \subset \mathbb{R}$ such that $P(X^{-1}(E) \cap X^{-1}(F)) = P(X^{-1}(E))P(X^{-1}(F))$. True/**False**

   sol. To show the independence, we have to show the equation *for all* $E, F \subset \mathbb{R}$.

Problems - Need justification. No justification means **zero**!

1. Let E and F be the events that a family of 3 children has children of both sexes and has at most one boy, respectively. Are E and F independent? (5pts)

   sol. There are $2^3 = 8$ possible outcomes for the sexes of children (if we assume that there are only two sexes). Note that the situation is ELOP. The size of the event E is $2^3 - 2 = 6$, since there are two cases where the all of the children have same sex. So $P(E) = 6/8 = 3/4$. For F, there are two possible cases: no boys or exactly one boy. Each has 1 and $\binom{3}{1} = 3$ possibilities, so $|F| = 4$ and $P(F) = 4/8 = 1/2$. For the intersection, we only allow the cases where there is exactly one boy, which has 3 cases and so $P(E \cap F) = 3/8$. From this calculation, we have $P(E \cap F) = 3/8 = P(E)P(F)$, so E and F are *independent*.

2. Suppose that a 6-sided fair die is rolled twice. Compute the probability mass function of the random variable

   $$X = \text{the difference of the two rolls.}$$

   (5pts)

   sol. First, the range of X is $\{0, 1, 2, 3, 4, 5\}$. The number of possible outcome is $|\Omega| = 6^2 = 36$, and the situation is ELOP.

   (a) If the difference is 0, it means that the two rolls are same, i.e. we have 6 possible cases $\{(1, 1), (2, 2), \ldots, (6, 6)\}$ and $P(X = 0) = 6/36 = 1/6$.

   (b) If the difference is 0, we have $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$ and reverses of them, so $P(X = 1) = 5 \cdot 2/36 = 5/18$.

   (c) If the difference is 1, we have $\{(1, 3), (2, 4), (3, 5), (4, 6)\}$ and reverses of them, so $P(X = 2) = 4 \cdot 2/36 = 4/9$.

   Similarly, we can check that $P(X = k) = (6 - k) \cdot 2/36 = (6 - k)/18$ for $1 \leq k \leq 5$.