MATH 10B with Prof. Stankova
DIS 102; TTh 8:00~ 9:30am
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Student:
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Tue 3/12/19

Quiz 7

True/False - No explanation needed. (2pts)

1. Two Bernoulli trials are independent only if the probability of success and failure are each $\frac{1}{2}$. True/**False**

   sol. Bernoulli trials are independent or dependent of each other regardless of specific probabilities p of success and $1-p$ of failure. Example: roll a fair die for two times and let showing 6 be success. Two rolls are independent of each other but the probability for success is 1/6.

2. For any random variable X, the expected value $E(X)$ is always finite. True/**False**

   sol. No. For example, think about the HW 16 Problem 4, the St. Petersburg's paradox.

Problems - Need justification. No justification means **zero**!

1. You roll two fair 6-sided die 100 times. Let X be the number of times you roll a sum of 4. Identify the name of the distribution of X and find $P(X = 10)$. (5pts)

   sol. X follows a *binomial distribution*. For each try, we roll two dies and have $6^2 = 36$ possible outcomes. Among them, we have 3 cases that sum gives 4 $(1 + 3 = 2 + 2 = 3 + 1)$. Hence the probability for success is $p = 3/36 = 1/12$ and $n = 100$. So we get

$$P(X = 10) = \binom{100}{10}\left(\frac{1}{12}\right)^{10}\left(\frac{11}{12}\right)^{90}.$$

2. Assume that the random variable X follows Poisson distribution. If $E(X) = 1$, what is the probability $P(X \geq 1)$? (5pts)

   sol. If X follows Poisson distribution with the intensity λ, then $E(X) = \lambda = 1$. Hence $\lambda = 1$ and

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\lambda^0 e^{-\lambda}}{0!} = 1 - e^{-1}.$$