True/False - No explanation needed. (2pts)

1. For any random variable X, we have $E[X^2] \geq E[X]^2$. True/False

2. According to the Law of Large Numbers, the probability $P(|\overline{X} - \overline{\mu}| > \epsilon)$ decreases as n grows. True/False

Problems - Need justification. No justification means **zero**!

In this problem, we will approximate the following summation

$$\sum_{k=45}^{55} \binom{100}{k} \frac{1}{2^{100}}$$

using the standard normal distribution and the Central Limit Theorem. Let $X_1, \ldots, X_{100}$ be the Bernoulli variables with $p = 1/2$. Let $\overline{X} = \frac{X_1 + \cdots + X_{100}}{100}$ be the average, and let Z be the normalized RV corresponding to $\overline{X}$.

1. Find the range of $\overline{X}$, $\overline{\mu}$, and $\overline{\sigma}$. (3pts)

2. Explain why $P(|\overline{X} - \overline{\mu}| \leq \frac{1}{20})$ is same as the above summation. (3pts)

3. Using the given part of the standard normal table, approximate $P\left(|\overline{X} - \overline{\mu}| \leq \frac{1}{20}\right)$. (4pts)

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513