MATH 10B with Prof. Stankova
DIS 106; TTh 9:30~11:00
GSI: Seewoo Lee

Student:
SID:
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Quiz 8

True/False - No explanation needed. (2pts)

1. The z-score that we can use to compute $P(0 \le \overline{X} \le \overline{\mu} + z\overline{\sigma})$ can be equivalently seen in the probability $P(0 \le Z \le z)$. True/**False**

   sol. The inequality $0 \le \overline{X} \le \overline{\mu} + z\overline{\sigma}$ is equivalent to $-\frac{\overline{\mu}}{\overline{\sigma}} \le Z \le z$, so the probability $P(0 \le \overline{X} \le \overline{\mu} + z\overline{\sigma})$ is same as $P(-\frac{\overline{\mu}}{\overline{\sigma}} \le Z \le z)$.

2. If two random variables X and Y are independent, then $\mathrm{Var}[X+Y] = \mathrm{Var}[X-Y]$. **True**/False

   sol. Since X and Y are independent, we have $\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$. Similarly, we have $\mathrm{Var}[X-Y] = \mathrm{Var}[X+(-Y)] = \mathrm{Var}[X] + \mathrm{Var}[-Y] = \mathrm{Var}[X] + (-1)^2\mathrm{Var}[Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$. So $\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] = \mathrm{Var}[X-Y]$.

Problems - Need justification. No justification means **zero**!

1. Let X_1, X_2 be independent geometric random variables with probabilities p_1 and p_2, respectively. Let $Y = X_1 + X_2$. Find $\mathrm{Cov}(X_1, 2Y)$. (5pts)

   sol. By definition and the properties of expected value, we have

$$\begin{aligned}\mathrm{Cov}(X_1, 2Y) &= E[X_1 \cdot 2Y] - E[X_1]E[2Y] \\ &= 2E[X_1Y] - 2E[X_1]E[Y] \\ &= 2(E[X_1(X_1+X_2)] - E[X_1]E[X_1+X_2]) \\ &= 2(E[X_1^2 + X_1X_2] - E[X_1]^2 - E[X_1]E[X_2]) \\ &= 2(E[X_1^2] + E[X_1X_2] - E[X_1]^2 - E[X_1]E[X_2])\end{aligned}$$

   Since X_1 and X_2 are independent, we have $E[X_1X_2] = E[X_1]E[X_2]$. Hence the two terms cancel out and

$$\mathrm{Cov}(X_1, 2Y) = 2(E[X_1^2] - E[X_1]^2) = 2\mathrm{Var}[X_1] = \frac{2(1-p_1)}{p_1^2}.$$

2. Using the given part of the standard normal table, find the value of $P(1.7 \leq X \leq 2.5)$, where X follows the normal distribution with $E[X] = 1$ and $\mathrm{Var}[X] = 4$. (5pts)

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513

sol. We have $\mu = 1$ and $\sigma = \sqrt{4} = 2$. Then the normalized random variable

$$Z = \frac{X-1}{2}$$

will follow the standard normal distribution. We have

$$1.7 \leq X \leq 2.5 \Leftrightarrow 0.7 \leq X - 1 \leq 1.5 \Leftrightarrow 0.35 \leq \frac{X-1}{2} \leq 0.75,$$

so $P(1.7 \leq X \leq 2.5) = P(0.35 \leq Z \leq 0.75) = P(0 \leq Z \leq 0.75) - P(0 \leq Z \leq 0.35)$. If you see the table, this is $0.2734 - 0.1368 = 0.1366$.