MATH 10B with Prof. Stankova
DIS 102; TTh 8:00~ 9:30am
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Quiz 9

Student:
SID:
Tue 4/2/19

True/False - No explanation needed. (2pts)

1. A PDF $f(x)$ cannot have values greater than 1. True/**False**

   sol. It can. For example, consider $f(x) = 3x^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise. However, the integral $\int_a^b f(x)dx = P(a \leq X \leq b)$ cannot have values greater than 1.

2. There is a distribution fails to have a well-defined mean μ, but has a well-defined median m. **True**/False

   sol. Pareto distribution (in the HW problem) is such a distribution.

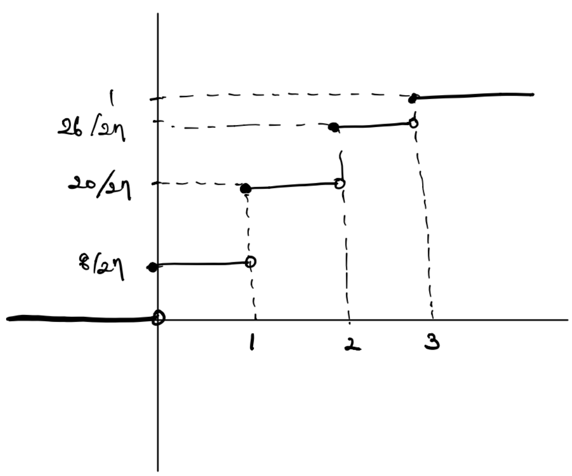
Problems - Need justification. No justification means **zero**!

1. Let X be a binomial distribution with $n = 3$ and $p = 1/3$. Find CDF of X and draw a graph of it. (5pts)

   The range is $R_X = \{0, 1, 2, 3\}$ and PMF is

$$f(k) = P(X = k) = \binom{3}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{3-k}.$$

   By definition, CDF is $F(x) = P(X \leq x) = \sum_{k \leq x} f(k)$ which looks as the following.



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2. Let $F(x)$ be a CDF defined as

$$F(x) = \begin{cases} 1 - e^{-x^2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find a corresponding PDF and compute $P(X \geq 1)$. (5pts)

sol. To compute the corresponding PDF, we differentiate the CDF:

$$F(x) = \begin{cases} (1 - e^{-x^2})' = -(-2x)e^{-x^2} = 2xe^{-x^2} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

(It doesn't matter whether you use $x \geq 0$ or $x > 0$ - both are correct.) The probability is

$$P(X \geq 1) = 1 - P(X < 1) = 1 - F(1) = e^{-1}.$$