MATH 10B with Prof. Stankova
DIS 106; TTh 9:30∼11:00
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Quiz 9

True/False - No explanation needed. (2pts)

1. If a PDF $f(x)$ is 0 outside interval $[a, b]$, we do not need to use improper integrals when working with its associated continuous RV. **True**/False

2. Both mean and the median of an exponential distribution directly depend on the initial condition $f(0)$ of DE $f'(x) = Cf(x)$ and on nothing else. **True**/False

Problems - Need justification. No justification means **zero**!

1. Find $c \in \mathbb{R}$ such that the following function defines a PDF

$$f(x) = \begin{cases} cx(3-x) & 0 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(5pts)

sol. To be a PDF, it should satisfy $\int_{-\infty}^{\infty} f(x)dx = 1$. Since $f(x) = 0$ for $x > 3$ or $x < 0$, the integral is same as the integral over $0 \le x \le 3$. We have

$$1 = \int_0^3 cx(3-x)dx = c\int_0^3 3x - x^2 dx = c\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{9c}{2},$$

so $c = \frac{2}{9}$. For such c, $f(x)$ is piecewise continuous and nonnegative. (You have to check this!)

2. Calculate the **mean** and **median** of the following PDF

$$f(x) = \begin{cases} \frac{2}{(1+x)^2} & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(5pts)

sol. The mean μ is

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 \frac{2x}{(1+x)^2}dx = \int_1^2 \frac{2(u-1)}{u^2}du = \left[2\ln u + \frac{2}{u}\right]_1^2 = 2\ln 2 - 1.$$

The median m is

$$\frac{1}{2} = \int_{-\infty}^{m} f(x)dx = \int_0^m \frac{2}{(1+x)^2}dx = \int_1^{m+1} \frac{2}{u^2}du = \left[-\frac{2}{u}\right]_1^{m+1} = 2 - \frac{2}{m+1},$$

so we get $m = \frac{1}{3}$.