1. (Pigeon hole principle) In the US (300 million people), everyone likes one of 10 different colors. Show that there exist at least 5 people that like the same color, have the same three letter initial, and have the same birthday.

2. (Induction) (a) Prove that for any $n \geq 1$, $n^3 - n$ is divisible by 2.
(b) Prove that it is also divisible by 3.
(c) Conclude that $n^3 - n$ is divisible by 6 for any $n \geq 1$.

3. How many ways can you line up 4 couples if each couple needs to stand next to each other?

4. (Binomial theorem) (a) What is a coefficient of x^7y^9 in the expansion of $(2x - 5y)^{16}$?
(b) What is a coefficient of x^2 in the expansion of $(2x + \frac{1}{3x})^{12}$?

5. (a) How many positive integers from 1 to 1000 are divisible by 3 but not by 4?
(b) How many positive integers from 1 to 1000 are divisible by 4 or 5?

6. I pick 3 numbers a, b, c (not necessarily different) from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability that $ab + c$ is even?

7. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 20$ with $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4$?

8. (Balls and urns) (a) How many ways we can put 20 distinct balls into 5 identical urns?
(b) 20 identical balls into 5 identical urns?
(c) 20 distinct balls into 5 distinct bins?
(d) 20 identical balls into 5 distinct bins?

Challenges

1. Let $f(x) = e^{-x^2}$. Prove that for any $n \geq 1$, there exists a polynomial $p_n(x)$ such that
$$\frac{d^n}{dx^n} f(x) = p_n(x) e^{-x^2}.$$
Also, show that the degree of $p_n(x)$ is n, and the coefficient of x^n is $(-2)^n$.

2. How many solutions are there to the equation $x_1 + x_2 + x_3 = 20$ with $0 \leq x_1, x_2, x_3 \leq 8$?