Final review 2

1. Given the following statistics, what is the probability that a woman has cancer if she has a positive mammogram result?
   - One percent of women over 50 have breast cancer.
   - Ninety percent of women who have breast cancer test positive on mammograms.
   - Eight percent of women will have false positives.
2. I roll a fair 6-sided die over and over again until I roll a 6. What is the probability that it takes me more than 10 tries? What is the expected number of total rolls I need and what is the variance?
3. In a class of 50 males and 90 females, I give out 4 awards randomly. What is the probability that females will win 2 awards if the awards must go to different people? What about if the same person can win multiple awards?
4. I roll a fair 6-sided die over and over again until I roll a 6. What is the probability that it takes me more than 10 tries? What is the expected number of total rolls I need and what is the variance?
5. While pulling out of a box of cookies, what is the expected number of cookies I have to pull out before I pull out an oatmeal raisin if 25% of cookies are oatmeal raisin and I choose with replacement? What is the variance?
6. Suppose that the average shopper spends 100 dollars during Black Friday, with a standard deviation of 50 dollars. What is the probability that a random sample of 100 shoppers will have spent more than \$3000? (You can use CLT to approximate the probability.)
7. Let
$$f(x) = \begin{cases} \frac{c}{x^4} & x \le -1 \\ 0 & \text{otherwise.} \end{cases}$$
   Find c such that $f(x)$ is a PDF. Graph f and the CDF F. Find the mean and median of $f(x)$.
8. Try to do sufficiently many exercises for statistics:
   - Find confidence interval
   - Maximum likelihood estimator
   - Hypothesis testing: Z-test, T-test, χ^2-test (Goodness of fit test & Independency test). Set H_0 and H_1, compute corresponding statistics and p-value using tables or online calculators, find critical value and rejection region, draw a conclusion.
   - Least squares

Challenges

1. Let X_1, X_2, X_3, X_4 be identical and independent Poisson random variables with $\lambda = 1$. Let $\overline{X}$ be the average of them. Compute $\mathrm{Var}[\overline{X}]$ and $\mathrm{Cov}(X_1, \overline{X})$.
2. We choose one number from the set $\{1, 2, 3, \ldots, n\}$ randomly. Assume that all the probabilities for each number are the same. Let X be a random variable which is the number we choose. Prove that $E[X^3] = nE[X]^2$ by induction on n.