Basics of counting

1. How many different three-letter initials are there that begin with an A? Contain an A?
2. How many 6-element RNA sequences
   (a) end with GU?
   (b) contain only A or U?
3. (a) How many different functions are there from a set with n elements to a set with m elements?
   (b) How many different injective functions are there from a set with n elements to a set with m elements? You may assume $n \leq m$.
4. How many positive integers between 100 and 999 inclusive
   (a) are divisible by 7?
   (b) are divisible by 3 or 4?
   (c) are divisible by 3 but not by 4?
   (d) if we change the range from 100∼999 to 100∼1000, how the answer for (b) will be changed?
5. How many strings of 5 decimal digits
   (a) do not have the same digit?
   (b) do not have two consecutive digits that are the same?
   (c) start with an odd digit?
   (d) end with an odd digit?
   (e) do not have the same digit and end with an odd digit?
6. How many diagonals does a convex polygon with n sides have?
7. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
   (a) the bride must be next to the groom?
   (b) the bride is not next to the groom?
   (c) the bride is positioned somewhere to the left of the groom?

Source: Rosen's *Discrete Mathematics and its Applications.*