Worksheet 0

Basics of counting

1. How many different three-letter initials are there that begin with an A? Contain an A? $1 \times 26 \times 26 = 676$, at least one A: $26^3 - 25^3 = 1951$, exactly one A: $3 \times 25 \times 25 = 1875$.
2. How many 6-element RNA sequences
   (a) end with GU? $4^4 = 256$
   (b) contain only A or U? $2^6 = 64$
3. (a) How many different functions are there from a set with n elements to a set with m elements? m^n
   (b) How many different injective functions are there from a set with n elements to a set with m elements? You may assume $n \leq m$. $m(m-1)\cdots(m-n+1)$.
4. How many positive integers between 100 and 999 inclusive
   (a) are divisible by 7? $\lfloor 900/7 \rfloor = 128$
   (b) are divisible by 3 or 4? $\lfloor 900/3 \rfloor + \lfloor 900/4 \rfloor - \lfloor 900/(3 \times 4) \rfloor = 300 + 225 - 75 = 450$
   (c) are divisible by 3 but not by 4? $\lfloor 900/3 \rfloor - \lfloor 900/(3 \times 4) \rfloor = 300 - 75 = 225$.
   (d) if we change the range from 100∼999 to 100∼1000, how the answer for (b) will be changed? $\lfloor 901/3 \rfloor + \lfloor 901/4 \rfloor + 1 - \lfloor 901/(3 \times 4) \rfloor = 300 + 225 + 1 - 75 = 451$
5. How many strings of 5 decimal digits
   (a) do not have the same digit? $10 \times 9 \times 8 \times 7 \times 6 = 30240$
   (b) do not have two consecutive digits that are the same? $10 \times 9 \times 9 \times 9 \times 9 = 65610$
   (c) start with an odd digit? $5 \times 10 \times 10 \times 10 \times 10 = 50000$
   (d) end with an odd digit? $10 \times 10 \times 10 \times 10 \times 5 = 50000$
   (e) do not have the same digit and end with an odd digit? $5 \times 9 \times 8 \times 7 \times 6 = 15120$

   Note. Strings of 5 decimal digits contain strings that starts with 0, for example, 00254.
6. How many diagonals does a convex polygon with n sides have? $n(n-3)/2$
7. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
   (a) the bride must be next to the groom? $2 \times 5 \times 4! = 240$
   (b) the bride is not next to the groom? $6! - 240 = 480$
   (c) the bride is positioned somewhere to the left of the groom? $6!/2 = 360$

Source: Rosen's *Discrete Mathematics and its Applications*.