

More Counting

1. How many six character strings of English letters

i) contain an A ? $26^6 - 25^6$

ii) contain an E or an I ? $26^6 - 24^6$

iii) start with a C or start with a D ? $2 \cdot 26^5$

iv) start with a C or end with a D ? $26^5 + 26^5 - 26^4$

v) contain exactly one X ? $6 \cdot 25^5$

2. Consider the set $\{1, 2, 3, \dots, 99\}$.

i) How many subsets are there that contain the numbers 1 or 99? $2^{98} + 2^{98} - 2^{97}$

ii) How many subsets contain only even numbers? 2^{49}

iii) How many subsets only contain numbers divisible by 3 but not divisible by 4? 2^{33-8}

iv) How many subsets only contain numbers divisible by 4 or divisible by 6? 2^8

v) (***) How many subsets are there that have an even number of elements? 2^{98}

3. How many different ways could a Best of Seven series go?

$$2 \cdot (1 + 4 + 10 + 20) = 70$$

4. How many binary strings are there of length 7 with

i) two consecutive zeroes in the beginning or two consecutive ones at the end? $2^5 + 2^5 - 2^3$

ii) a one in the first, fourth, or last digit? $2^6 + 2^6 + 2^6 - 2^5 - 2^5 - 2^5 + 2^4$

iii) exactly two zeroes? $6 + 5 + 4 + 3 + 2 + 1 = 21$

iv) an even number of ones? 2^6

v) (***) exactly one pair of adjacent zeroes? 38

5. (*) Suppose students at a school are able to take French, Latin, Spanish (or any combination of the three). There are 500 students; 200 students take French, 150 students take Latin, and 170 students take Spanish. Furthermore, you also know that 50 students take French and Latin, 85 students take French and Spanish, and 70 students take Spanish and Latin. If there are 150 students who take no language course, how many students take all three languages? 35

6. How many ways are there to split 10 people into a Red Team and Blue Team? The teams do not have to be even, but each side needs at least one person. $2^{10} - 2$

(***) What if we want to have a Green Team as well? $3^{10} - (2^{10} + 2^{10} + 2^{10}) + 3$

7. How many people must you talk to in order to guarantee you find two of them born in the same month? $12 + 1 = 13$

8. Suppose in the month of January (31 days) you make a resolution to go to the gym 100 times. However, the gym won't let you go more than 3 times a day. Can you meet your resolution? What is the closest you can get to your goal? **No. If you go to the gym everyday for 3 times a day, then you can go $3 \cdot 31 = 93$ times.**
9. Suppose you have a drawer of socks in a dark room. If you have just white and black socks, how many socks do you need to grab in order to ensure you get a matching pair? **$2 + 1 = 3$**
What if you have 7 different colors of socks? **$7 + 1 = 8$**
What if you have 3 legs and 5 different colors of socks? **$2 \cdot 5 + 1 = 11$**
What if you have n legs and k colors of socks? **$(n - 1)k + 1$**
10. (★★) Is it always true that in a group of 5 people, two of them will have the same number of friends within the group? **Yes. For each person, she/he may have 0, 1, 2, 3, or 4 friends among other people in the group. Now assume that there are no people who have the same number of friends. This means that one of them (let's call A) have 0 friends, and some other (let's call B) have 4 friends - which is impossible! (A can't be friend with any other else, while B is friend with everyone in that group. Then A and B can and can't be friend.) Hence, if we consider 5 people as 5 pigeons and possible number of friends as holes, there are at most 4 holes, so there exist two pigeons who are in the same hole. In other words, there exist two people with same number of friends.**