More combinations and permutations

1. How many permutations of the letters $ABCDEFG$ contain:

(a) the string ACE?

(b) the strings AG and FCB?

(c) the strings AB, DC, and GE?

(d) the strings ACB and GFE?

2. Ten women and eight men are on the faculty of a mathematics department at a school.

(a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?

(b) How many ways are there to select a committee of five members of the department if at least one man and at least one woman must be on the committee?

Binomial coefficients

3. What is the coefficient of x^6y^{10} in the expansion of $(2x + 5y)^{16}$?
4. The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}, 0 \leq k \leq 10$, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately preceding this row in Pascal's triangle.

5. Show that if n and k are integers with $1 \leq k \leq n$, then $\binom{n}{k} \leq n^k/2^{k-1}$.
6. Prove the hockeystick identity:

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

whenever n and r are positive integers.

More counting

7. How many ways are there to distribute

(a) 10 distinguishable balls into four distinguishable bins?

(b) 10 indistinguishable balls into four distinguishable bins?

8. How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 14 coins in it?
9. How many solutions are there to the inequality $x_1 + x_2 + x_3 + x_4 \leq 15$?
10. In the Chinese game Tractor, 48 of the 54 cards of a standard deck (counting jokers) are dealt to four players. How many different ways are there to deal Tractor hands to the four players?