Worksheet 6

Algorithms

1. Demonstrate bubble sort to sort the list 3, 4, 2, 1.
2. Demonstrate the quick sort to sort the list $3, 6, 2, 5, 1, 4$.
3. Demonstrate the stable matching algorithm when men and women have the preferences $m_1 : w_1 > w_2, m_2 : w_1 > w_2$ and $w_1 : m_1 > m_2, w_2 : m_1 > m_2$.
4. Three women A, B, C are proposing to men E, F, G. Their preferences are as follows:

A	B	C	E	F	G
$E > G > F$	$E > G > F$	$G > E > F$	$C > A > B$	$A > B > C$	$B > C > A$

   Show the stable matching algorithm with the women proposing to the men by clearly showing all rounds in a table.

5. Sort the list $2, 1, 6, 4, 5, 3$ using both bubble sort and quicksort.

Inductions

6. Prove using mathematical induction that for all $n \geq 1$,

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}.$$

7. Prove that

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n - 1)(2n + 1)} = \frac{n}{2n + 1}.$$

8. (*) Prove using mathematical induction that for all $n \geq 1$, $6^n - 1$ is divisible by 5.
9. Let $\{a_n\}_{n \geq 1}$ be a sequence defined as $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$. Prove that $a_n \leq 2$ for all $n \geq 1$, by using mathematical induction.