I. Bayes' Rule

1. Suppose a new cancer test has a 95% chance of correctly identifying that a sick patient has cancer and a 10% chance of incorrectly identifying that a healthy patient has cancer. Assume that 5% of the population has this form of cancer. Compute the following probabilities:
   a) The probability that the test identifies a randomly chosen person as having cancer.
   b) The probability that a person who tests positive for cancer actually has cancer.
   c) The probability that a person who tests negative for cancer does not have cancer.
   d) The probability that the test gives an incorrect result.

2. Suppose a weatherman predicts rainy days correctly with an accuracy of 90% and predicts clear days correctly with an accuracy of 90%. Given that it rains 20% of the time, and the weatherman predicted that it will rain tomorrow, what is the probability that it will actually rain tomorrow?

II. Review

1. How many ways can you arrange 10 marbles in a row if 4 are red, 3 are blue, and 3 are green (marbles of the same color are identical)?

2. How many ways are there to arrange 12 ones and 18 zeros in a line if every one must be immediately followed by a zero?

3. Show that in a group of 10 people, each of whom is friends with at least one of the others, there are two people with the same number of friends (within the group).

4. Prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$ using a combinatorial argument and by algebraic manipulation.

5. How many ways are there to split 6 distinguishable people into 3 distinguishable non-empty teams?

6. How many solutions are there to the equation $x + y + z = 12$ such that $1 \leq x \leq 4$?

7. Let $\{a_n\}_{n\geq 0}$ be the sequence defined by $a_0 = 1$ and $a_{n+1} = 4a_n + 1$. Prove that $a_n = \frac{4^{n+1}-1}{3}$.