

Discussion on Probability and Independence

1. Concept of Independence
 - What is the probability that two people chosen at random were born during the same month of the year?
 - What is the probability that in a group of n people chosen at random, there are at least two born in the same month of the year?
 - How many people chosen at random are needed to make the probability greater than $1/2$ that there are at least two people born in the same month of the year?
2. Two dice were rolled. Are the events that the first die rolled is a 1 and that the sum of the two dice is a 7 independent?
3. Find the smallest number of people you need to choose at random so that the probability that at least two of them were both born on April 1st exceeds $1/2$.
4. Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has
 - exactly three boys?
 - at least one boy?
 - at least one girl?
 - all children of the same sex?
5. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?
6. Let E and F be the events that a family of n children has children of both sexes and has at most one boy, respectively. Are E and F independent if
 - $n = 2$
 - $n = 4$
 - $n = 5$