I. Random variables

1. Suppose that we roll two die and let X be equal to the maximum of the two rolls. Find $P(X \in 1, 3, 5)$ and draw the PMF for X.
Solution: First we draw the PMF. We calculate $P(X = x)$ by counting the number of ways we can roll two die so that the maximum is x and then dividing by the total number of possibilities, which is 36. So for instance, the only way to get $X = 1$ is if we roll (1, 1) and hence $P(X = 1) = 1/36$. Then $P(X = 2) = f(1, 2); (2, 2); (2, 1)/36 = \frac{3}{36}$. Thus, we have that:
$f(1) = \frac{1}{36}, f(2) = \frac{3}{36}, f(3) = \frac{5}{36}, f(4) = \frac{7}{36}, f(5) = \frac{9}{36}, f(6) = \frac{11}{36}$
We draw the PMF with stalks at 1 through 6 of those respective heights. Then $P(X \in 1, 3, 5) = P(X = 1) + P(X = 3) + P(X = 5) = 5/12$

2. When rolling two die, let Y be equal to the first die roll. Are X, Y independent random variables?
Solution: No. Intuitively, if we know that the first die roll is a 6, then the maximum has to be a 6. Mathematically writing that, we see that$P(X = 6, Y = 6) = P(Y = 6) and P(X = 6) \neq 1$ so $P(X = 6, Y = 6) \neq P(Y = 6)P(X = 6)$

3. I flip a fair coin 4 times. Let X be the number of heads I get. Draw the PMF for X.
Solution: The range is $0, 1, 2, 3, 4$. Then $P(X = x)$ is the number of ways to get x heads over the total number of ways so $P(X = x) = \frac{\binom{4}{x}}{2^4}$.

4. I roll two fair four sided die with sides numbered $1 - 4$. Let X be the product of the two numbers rolled. Find the range of X and draw the PMF for X.
Solution: The range is all products of two numbers in 1, 2, 3, 4. This is 1, 2, 3, 4, 6, 8, 9, 12, 16. We calculate:

x	1	2	3	4	6	8	9	12	16
$f(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

5. I draw 5 cards from a deck of cards. Let X be the number of hearts I draw. What is the range of X and draw the PMF of X. Use this to find the probability that I draw at least 2 hearts.
Solution: The range is 0,1,2,3,4,5. To calculate $f(x) = P(X = x)$, we count the number of good ways over the total number of ways. The number of good ways to draw x hearts is to first pick out x hearts out of the 13 hearts, and then fill out the rest of the hand and pick 5 - x non-heart cards from the remaining 39 cards. Thus $f(x) = \frac{\binom{13}{x}\binom{39}{5-x}}{\binom{52}{5}}$. Thus we have that $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = \frac{\binom{13}{2}\binom{39}{3} + \binom{13}{3}\binom{39}{2} + \binom{13}{4}\binom{39}{1} + \binom{13}{5}\binom{39}{0}}{\binom{52}{5}}$

II. Binomial distribution

1. A coin is biased so that the probability of heads is 2/3. What is the probability that exactly four heads come up when the coin is flipped seven times, assuming that the flips are independent.
Solution: $X\ Bin(7, 2/3)$, $P(X = 4) = \binom{7}{4}(\frac{2}{3})^4(\frac{1}{3})^3 = 560/2187$

2. Suppose that the probability that a 0 bit is generated is 0.9, that the probability that a 1 bit is generated is 0.1, and that bits are generated independently. What is the probability that exactly eight 0 bits are generated when 10 bits are generated.
   Solution: $X\ Bin(10, 0.9)$, $P(X = 8) = \binom{10}{8}(0.9)^8(0.1)^2 \approx 0.1937$