

Expected Value and Variance

1. If Z is some random variable, how do you find the *expected value* of Z ?

How do you find the *variance* of Z (two ways)?

2. Suppose you roll two, fair 4-sided dice. Let X denote the minimum of the two rolls and Y the maximum of the two rolls. Are these two random variables *independent*?

For the above random variables, find the following:

- a) The expected value of X , $E(X)$
 - b) $E(Y)$
 - c) $E(2X + 3Y)$
 - d) $E(XY)$
 - e) The variance of X , $\text{Var}(X)$
 - f) $\text{Var}(2Y)$
3. You have a weighted coin that lands on tails twice as often as it lands on heads. If you flip this coin 10 times, what is the expected number of heads you get? What is the standard error of this random variable?
 4. Someone flips a biased coin an unknown number of times. They tell you that the average number of heads is 10 with a variance of 5. Can you figure out both the chance that the coin lands on heads *and* the number of times the coin was flipped?
 5. Suppose you are drawing cards from a standard deck of 52 cards. What is the expected number of cards you will need until you draw the Ace of Spades?
(★) What is the variance of the number of cards until you get the Ace of Spades?
 6. On average, your friend has a 10% of making a certain trick shot. If you agree to let them shoot until they make it, how many tries do you expect them to take? What is the standard error of this random variable?
(★★) How many tries would you expect them to need to make the shot twice?
 7. Suppose you draw 5 cards from a standard deck of 52 cards. What is the expected number of queens you get? What is the standard error of the number of queens that you get?
 8. You observe that a radioactive sample releases an emission on average once every ten seconds. What is the probability that there is at least one emission over the course of a minute? What is the expected number of emissions in a minute? What is the standard error for the number of emissions in a minute?