Worksheet 18

Mean, Median and Variance of Continuous RVs

1. For each PDF, calculate the mean and the median and the variance:
   (a) $f(x) = x^{-2}$ for $\frac{1}{2} \leq x \leq 1$ and $f(x) = 0$ otherwise.
   (b) $f(x) = x^2(2x + \frac{3}{2})$ for $0 < x \leq 1$ and $f(x) = 0$ otherwise.
   (c) $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$ and $f(t) = 0$ otherwise.
   (d) $f(x) = c(1 - x^2)$ for $-1 < x < 1$ and $f(x) = 0$ otherwise. Does the answer depend on c? Why?

2. **Chromosomal recombination** is a process by which two chromosomes join together and exchange DNA. The point along the DNA at which the join occurs is randomly located. Suppose X is a RV denoting the location with $0 \leq X \leq 2$. In an experiment, $E[X] = 1$ and $Var[X] = \frac{1}{3}$. Are the findings consistent with the hypothesis that all locations along the chromosome are equally likely to contain the join point? Explain.

Midterm 2 Review

1. Find a, b or c given the PDF, then find the CDF.
   (a) $f(x) = c(1 - x^2)$ for $-1 < x < 1$ and $f(x) = 0$ otherwise.
   (b) $f(x) = c/x^2$ for $x > 10$ and $f(x) = 0$ otherwise.
   (c) $f(x) = a + bx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise, given $E(X) = \frac{3}{5}$.

2. Suppose you take a random sample of 10 tickets without replacement from a box containing 20 red tickets and 30 blue tickets.
   (a) What is the chance of getting exactly 4 red tickets?
   (b) Repeat (a) for sampling with replacement.

   Hint: identify the distributions first.

3. Suppose that we observed 10 frogs in a pond during the obsevation period of 100 days. Find the Poisson approximation to the probability of observing $X = k$ frogs each day. Using that approximation to calculate the probability that
   (a) you observe precisely one frog today?
   (b) you observe more than one frog today?
   (c) you observe no frogs today?

4. Suppose that a test for opium use has a 2% false positive rate and a 5% false negative rate. That is, 2% of people who do not use opium test positive for opium, and 5% of opium users test negative for opium. Furthermore, suppose that 1% of people actually use opium.
   (a) Find the probability that someone who tests negative for opium use does not use opium.
   (b) Find the probability that someone who tests positive for opium use actually uses opium.

Source: some from Stewart's *Biocalculus*, the others from internet.