

Bounding Probabilities

Simple intuition:

1. Draw the normal pdf. Highlight the portion of the pdf capturing $\{|X - \mu| \geq k\sigma\}$ for $k = 0.5, 1, 2, 5$, roughly.
2. What are some reasons Chebyshev may be lossy? What are some reasons it may be sharp?

Calculations:

1. Suppose X is now Poisson with parameter λ . What are μ and σ for this distribution?
 - (a) Compute $\mathbb{P}[|X - \mu| \geq 2 \cdot \sigma]$.
 - (b) Compute $\mathbb{P}[|X - \mu| > 2 \cdot \sigma]$.
 - (c) Approximate $\mathbb{P}[|X - \mu| > 2 \cdot \sigma]$ using Chebyshev.
 - (d) Approximate $\mathbb{P}[|X - \mu| \leq 0.5 \cdot \sigma]$ using Chebyshev.
2. Suppose that X has Laplace distribution with mean 0, i.e. its pdf is

$$f(x) = \frac{1}{2}e^{-|x|}.$$

Note that the variance of this distribution is 2.

- (a) Compute $\mathbb{P}[|X| > 4]$.
- (b) Compute $\mathbb{P}[|X| \geq 4]$.
- (c) Use Chebyshev to approximate $\mathbb{P}[|X| > 4]$.

Source: Rosen's *Discrete Mathematics and its Applications*.