1. Maya shoots a basketball 100 times and makes 73 shots.

   (a) Assuming shots are independent, find a 95% confidence interval for the probability p of her making a single shot.

   A 95% confidence interval is given by

   $$\left(\hat{\mu} - 2\frac{\hat{\sigma}}{\sqrt{n}}, \hat{\mu} + 2\frac{\hat{\sigma}}{\sqrt{n}}\right)$$

   We estimate $\hat{\sigma}$ of the Bernoulli random variable as $\sqrt{.73(1-.73)} \approx .444$. Therefore as $n = 100$ and $\hat{\mu} = .73$, the interval is $(.686, .774)$

   (b) Estimate the variance of her making a shot using $\frac{1}{n-1}\sum_{k=1}^{n}(x_k - \overline{x})^2$

   Every shot that she shoots is either a 1 or 0. For the 73 shots she makes, $(x_k - \overline{x})^2 = (1-.73)^2 = .0729$. For the 27 misses, the value is $(.73)^2 = .5329$. Therefore

   $$s^2 = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \overline{x})^2 = \frac{1}{n-1}(73 \cdot .0729 + 27 \cdot .5329) \approx .199$$

2. Packer High School's track and field team averages 16 meters on their shot put throws, with a standard deviation of 1.7 meters. Assuming throws are normally distributed, what is the probability that an athlete throws less than 14 meters?

   Converting X to a standard normal, we see

   $$P(X < 14) = P(\frac{X-16}{1.7} < -1.18) = .5 - z(1.18) = 11.9\%$$

3. People visit Grimaldi's Pizzaria. Hour by hour, the number of people who visit is 11,5,3,5,4,8,5,4,2,9.

   (a) Find a 95% confidence interval for λ, the average number of people who visit in an hour. In this case λ is the average, which is 5.6. Therefore $\hat{\sigma} = \sqrt{5.6} \approx 2.37$. This makes our

   interval

   $$\left(\hat{\mu} - 2\frac{\hat{\sigma}}{\sqrt{n}}, \hat{\mu} + 2\frac{\hat{\sigma}}{\sqrt{n}}\right) = (4.10, 7.10)$$

   (b) Estimate the variance using $\frac{1}{n-1}\sum_{k=1}^{n}(x_k - \overline{x})^2$ $s^2 = 8.044$

4. An art auction house in Amsterdam's average sale is 3.2 million euros with a standard deviation of 800,000 euros. Assuming sales are normally distributed, what is the probability that a piece of art is sold for more than 5 million euros?

$P(X > 5) = P(\frac{X-3.2}{.8} > 2.25) \approx .5 - .4878 = 1.2\%$

5. The age of onset of multiple sclerosis is well described by a normal random variable with unknown mean and with standard deviation 7.6 years. The age of onset is measured for 32 individuals. Find the probability that the sample mean falls within 2 years of the true population mean.

The sample mean is a normal random variable with mean μ and standard deviation $7.6/\sqrt{32} \approx 1.34$. Therefore this probability is

$$\int_{\mu-2}^{\mu+2} \frac{1}{1.34\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2(1.34)^2}}$$

by substituting $u = \frac{x-\mu}{1.34}$ we get

$$\frac{1}{\sqrt{2\pi}} \int_{-2/1.34}^{2/1.34} e^{-u^2/2} du = 2z(1.49) \approx .864$$