I. Maximum Likelihood Estimation

1. Suppose you flip a coin 100 times and get 30 heads. Estimate the probability p that a single flip of the coin is a head and find 90% and 99% confidence intervals for p ...
   a) directly (using $\overline{x}$)
   b) using maximum likelihood estimation

2. Suppose a hospital records the number of critical patients they get per day over the course of 10 days, and get the following data: $10, 4, 3, 7, 5, 8, 2, 11, 12, 8$. Assume that the number of critical patients the hospital receives on any particular day is modeled by a Poisson distribution X with unknown parameter λ. Estimate λ using MLE.

3. Suppose X is a geometric random variable with unknown parameter p. You randomly sample X three times and get the values $5, 3, 8$. What is the MLE estimate for p given this data?

4. Suppose X is an exponential random variable with unknown parameter λ. You randomly sample X 5 times and get the values $25, 30, 33, 27, 31$. What is the MLE estimate for λ given this data?

5. Suppose X is a normal random variable with unknown mean and variance μ and σ^2. You randomly sample X 4 times and get the values $3, 4, 6, 7$. What is the MLE estimate for μ and σ^2 given this data?

6. For each of the above problems, determine whether the MLE estimate you obtained was biased or unbiased.