Chi-Squared Hypothesis Testing

1. You roll a die 60 times and get 10 1's, 10 2's, 10 3's, 2 4's, 8 5's, and 20 6's. Is the die fair? Use the significance level $\alpha = 0.05$.

   (a) What are H_0 and H_1?

   (b) Complete the following table.

value k	observed frequency n_k	expected frequency m_k	$(n_k - m_k)^2/m_k$
1	10		
2	10		
3	10		
4	2		
5	8		
6	20		

   (c) Calculate the χ^2 statistic and determine the number of degrees of freedom.

   (d) Draw a conclusion.

2. I claim that a coin is biased so that the probability of heads is 75%. When you flip the coin 40 times, you get 25 heads and 15 tails. Do you have enough evidence to reject my claim? Use the significance level $\alpha = 0.05$.

3. In a sample of 160 pea plants, we observe 120 tall purple plants, 30 tall white plants, 5 short purple plants, and 5 short white plants. Let the null hypothesis H_0 be that flower color and plant height are Mendelian traits. Let the alternative hypothesis H_1 be that flower color and plant height are not Mendelian traits. Using the significance level $\alpha = 0.05$, do we have enough evidence to reject H_0? (Recall that we expect the proportion of the four possible phenotypes (TP, TW, SP, SW) to be 9:3:3:1 if flower color and plant height are Mendelian.)

4. [HW34#1] We roll two 6-sided dice 100 times and record the outcomes for the sum of the dice in the following table.

value	observed frequency	expected frequency
2	6	
3	10	
4	9	
5	13	
6	13	
7	12	
8	11	
9	10	
10	7	
11	5	
12	4	

Calculate the expected frequencies, given the null hypothesis H_0 that both dice are fair. Compute the χ^2 statistic for this data. What is the p-value? Do we have enough evidence to reject the null hypothesis?