

Partial derivatives & Differential equations

1. Compute partial derivatives $f_x, f_y, f_{xx}, f_{xy}, f_{yx}, f_{yy}$ of given functions, and check that $f_{xy} = f_{yx}$ holds.

(a) $f(x, y) = e^{xy}$

(b) $f(x, y) = \sin(2x + y)$

(c) $f(x, y) = x \ln y + y \ln x$

(d) $f(x, y) = x^y$

2. Let $z = \sin(x^2 + y)$ and $x = e^t, y = \frac{1}{t}$. Compute $\frac{dz}{dt}$.

3. (Wave equation) Let v be a fixed constant. The following partial differential equation

$$\frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

is called Wave equation. Prove that $y(x, t) = \sin(x - vt) + \sin(x + vt)$ is a solution of the Wave equation.

Some review

1. How many ways can you rearrange the letters in BERKELEY?
2. How many ways can we split 9 people into 3 distinguishable teams with 3 people each? What if the teams are indistinguishable?
3. What is the probability that a 5-card poker hand contains at least one ace?
4. The average number of chocolates sold by Trader's Joe is 150 chocolates per day. What is the probability that exactly 150 chocolates will be sold tomorrow?