Math 53 (Multivariable Calculus), Section 102 & 108
Week 1, Friday
Aug 26, 2022
For the other materials: seewoo5.github.io/teaching/2022Fall

1. What you have learned in Math 1A and 1B?

2. Sketch the following curves: $(-\infty < t < \infty)$

   (a) $x = 2t - 1, y = 3t + 1$

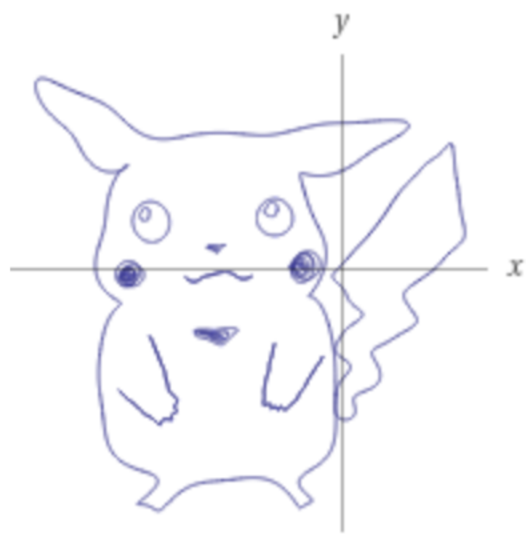
   (b) $x = e^t, y = e^{2t}$

   (c) $x = |\cos t|, y = |\sin t|$

   (d) $x = e^{-t}\cos t, y = e^{-t}\sin t$

3. Consider a parametrized curve $(x, y) = (f(t), g(t))$ parametrized by t. Could you explain a difference between it with another curve parametrized by $(x, y) = (f(2t), g(2t))$?

Here's a Pikachu curve for you:



Reference: https://www.wolframalpha.com/input?i=pikachu+curve

Solution

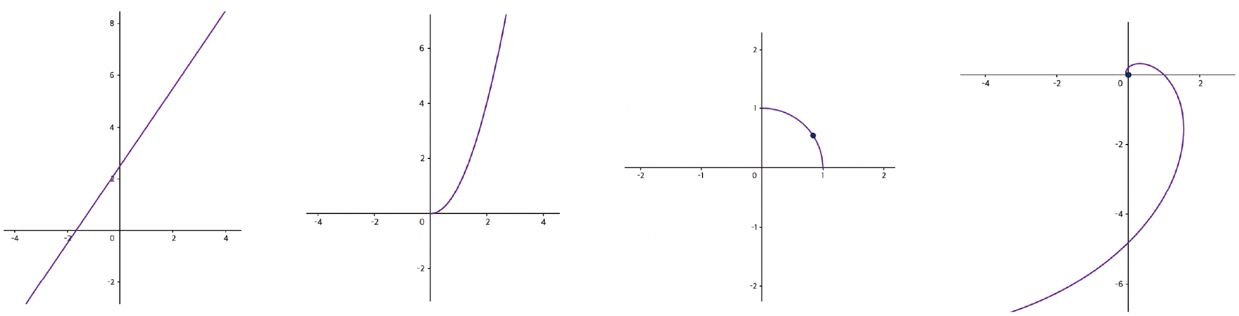
1. Single variable functions, limit and continuity, differentiation, integration, and their applications, ...

2. (a) Using $x = 2t-1 \Leftrightarrow t = (x+1)/2$, one can eliminate t and get a line $y = \frac{3}{2}(x+1)+1 = \frac{3}{2}x + \frac{5}{2}$.

   (b) We have $y = x^2$, but be careful - since $x = e^t$, we should have $x > 0$ and the curve will be the right half of the parabola (except the origin).

   (c) We have $x^2 + y^2 = 1$. However, both x and y should be non-negative, so the curve is the part of the unit circle on the first quadrant (including two endpoints $(1, 0)$ and $(0, 1)$).

   (d) Observe that $\sqrt{x^2 + y^2} = e^{-t}$ and $y/x = \tan t$. It is similar to a parametrization of a circle centered at origin $(\cos t, \sin t)$, and the slope of a line passes origin and a point are both equals to $\tan t$. Hence a point rotate around the origin in a counterclockwise direction. But the distance between (x, y) and the origin decreases exponentially as t increase. Hence, it is a spiral.



(a) (b) (c) (d)

3. Graphically they are the same - the second curve is traced twice times as fast.