

Math 53 (Multivariable Calculus), Section 102 & 108

Week 2, Friday

Sep 2, 2022

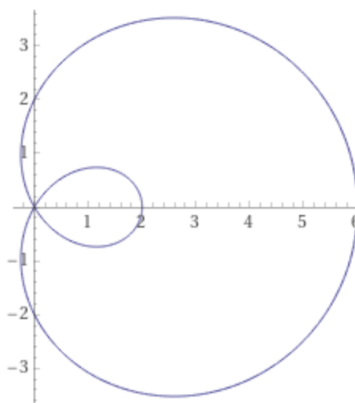
For the other materials: seewoo5.github.io/teaching/2022Fall

1. Find polar equations for the following curves.

(a) $x^2 - y^2 = 1$ (use $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$)

(b) $x^2 + (y - 1/2)^2 = 1/4$.

2. Assume that we have a following curve in polar equation $r = f(\theta)$.



Sketch the curves with the following polar equations.

(a) $r = \frac{1}{2}f(\theta)$

(b) $r = f(\theta + \pi/2)$

(c) $r = f(\pi - \theta)$

Solution

1. (a)

$$x^2 - y^2 = (r \cos \theta)^2 - (r \sin \theta)^2 = r^2(\cos^2 \theta - \sin^2 \theta) = r^2 \cos 2\theta = 1 \Leftrightarrow r^2 = \frac{1}{\cos 2\theta}$$

(b)

$$\begin{aligned}x^2 + \left(y - \frac{1}{2}\right)^2 &= r^2 \cos^2 \theta + \left(r \sin \theta - \frac{1}{2}\right)^2 \\&= r^2 \cos^2 \theta + r^2 \sin^2 \theta - r \sin \theta + \frac{1}{4} \\&= r^2 - r \sin \theta + \frac{1}{4} = \frac{1}{4} \\&\Leftrightarrow r = \sin \theta\end{aligned}$$

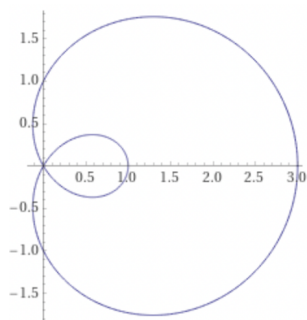
(Compare this to the Problem 3 of the worksheet for August 22nd.)

2. Note that the original curve's polar equation $r = 2 + 4 \cos \theta$.

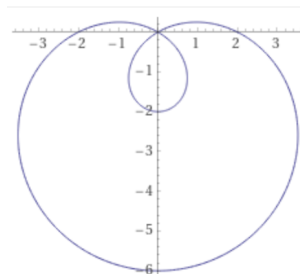
(a) It is shrunk by half.

(b) It is rotated by 90 degree, clockwise.

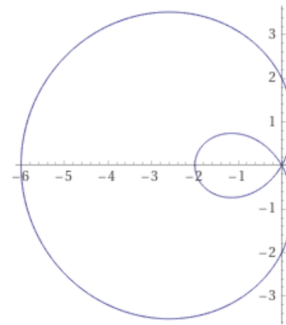
(c) It is reflected over the y axis.



(a)



(b)



(c)